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TFG TITLE: Improving the BADA 3 aerodynamic database for trajectory optimization

DEGREE: Bachelor's degree in Air Navigation Engineering

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Títol: Millora de la base de dades aerodinàmiques BADA 3 per la optimizació de trajectòries

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Resum

Tot i que l'aviació comercial s'ha desenvolupat intensament durant els últims anys, la optimització de trajectòries encara és un repte degut a les restriccions de seguretat, ambientals i de procediments. Per poder estudiar una trajectòria, i per tant optimitzar-la, primer de tot, s'ha de poder estudiar el comportament de l'aeronau. El comportament d'un avió ve donat per les equacions cinemàtiques i dinàmiques bàsiques, però evidentment, també és necessari conèixer el model aerodinàmic de l'aeronau.
Conscients d'aquest repte, EUROCONTROL ha desenvolupat un model de comportament d'aeronaus anomenat BADA (Base of Aircraft DAta) que conté el model de comportament, i per tant, aerodinàmic, d'un gran percentatge d'avions comercials actuals. Aquesta eina ha estat dissenyada per EUROCONTROL pels seus propis projectes de recerca però finalment ha estat posada a disposició del colectiu R+D.
El model BADA està format per un conjunt de fonaments teòrics en forma de polinomis genèrics utilitzat per calcular diferents paràmetres del comportament de les aeronaus. A més, ve acompanyat per un conjunt de sets de dades particulars per cada avió per poder particularitzar aquests polinomis a cada avió. Encara així, segueix sent un model genèric, i per tant, es pot entendre que no es del tot precís pel que la modelització de trajectòries requereix.
En aquest document es proposa una metodologia basada en la aproximació cinètica del model de comportaments d'aeronaus BADA i es millora fent servir un software anomenat *United States Air Force Stability and Control Digital DATCOM*. DATCOM és un software que implementa uns mètodes de càlcul d'estabilitat i control aerodinàmic desenvolupats al 1960 per la força aèria dels Estats Units.
Desprès de desenvolupar el model de dues de les aeronaus de llarg abast més comunes dins l'aviació comercial, aquest serà comparat amb el mateix model BADA i el model desenvolupat per Caroline Dietrich, estudiant de màster amb qui es col·labora en aquest projecte. La comparació es porta a terme amb un Boeing 767-300ER fent la modelització del creuer d'una trajectòria d'un vol Toronto - Los Ángeles.

Title : Improving the BADA 3 aerodynamic database for trajectory optimization

Author: Joaquim Villen

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Date: April 20, 2015

Overview

Although commercial aviation has developed strongly during recent years, the optimization of trajectories is still a challenge due to security, environmental and procedural restrictions. In order optimize an aircraft trajectory, and therefore, to model it, first of all, it is required to study the aircraft performance. The aircraft performance model is given by the basic kinematic and dynamic equations, although it is also necessary to know the aircraft aerodynamic model, i.e. its aerodynamic coefficients.

Aware of this challenge, EUROCONTROL has developed an aircraft performance model called BADA (Base of Aircraft Data) that contains the aircraft performance model for a large percentage of today's commercial aircraft. This tool has been designed by EUROCONTROL for their own research projects but has finally made available for the R& D collective.

The BADA model consists on a set of theoretical concepts in the form of generic polynomials used to calculate the aircraft performance. It also comes with a set of individual data sets for each plane to particularize these polynomials. But still, it remains a generic model, and therefore, it is not really accurate for what the trajectory optimization processes require.

This thesis proposes a methodology based on the kinetic approximation of the aircraft performance model and an improvement of BADA by using a software called United States Air Force and Stability Control Digital DATCOM. DATCOM is a software that implements calculation methods of aerodynamic stability and control developed in 1960 by the US Air Force.

After developing the model of two common long-haul aircraft, it will be compared with BADA model and a model developed by Ms. Caroline Dietrich, master student who worked as a researcher at École Polytechnique de Montréal. This comparison is performed by modeling the trajectory of a Boeing 767-300ER flight from Toronto to Los Angeles.

Tell me and I forget. Teach me and I remember. Involve me and I learn.
Benjamin Franklin (1706-1790)

CONTENTS

LIST OF FIGURES

ACKNOWLEDGMENTS

First of all, I wish to express my sincere thanks to my Bachelor Thesis Director, Mr David Saussié, as Assistant Professor at École Polytechnique de Montréal, not only his willing to exercise my thesis direction but also for sharing his expertise, helping me whenever I needed it and making this thesis possible.

I am also grateful to Mrs Caroline Dietrich, master graduate student and assistant researcher at École Polytechnique de Montréal, for her help and support.

I also place on record, the big help from Judit, who spent hours selflessly revising this thesis and helping me with all kind of errors.

I take this opportunity to express gratitude to all the wonderful people that I met during my amazing exchange year in Montréal. I would also make them part of this result.

Last, but not least, I really want to thank all my family and mostly my parents, for what I am today, and what they have done all this years. Without them, nothing of this would have been possible.

NOMENCLATURE

Unless otherwise specified this relation between identifier characters and meaning must be carried out to understand the thesis.

- D: aerodynamic drag
- m: aircraft mass
- T: thrust acting parallel to the aircraft velocity vector
- L: lift
- h: geodetic altitude
- g: gravitational acceleration
- V_{TAS} or V: true airspeed
- ρ: air density
- S: wing surface
- q: dynamic pressure
- b: wing span
- Re: Reynolds number
- M: Mach number
- γ: flight path angle
- ψ: aircraft heading
- T: temperature
- P: pressure
- ϕ: longitude
- λ: latitude

CHAPTER 1. INTRODUCTION

Within the commercial aviation world, aircraft fuel consumption is a research trending topic. This thesis focuses on the search for a more accurate aerodynamic model than current models for some specific aircraft. The main reason is that current models are not accurate enough for aircraft trajectory optimization. It will be useful to work on the trajectory optimization problem with the aim of reducing the airline costs related to fuel consumption. At the same time, the optimization will also contribute to a reduction of CO_2 atmosphere emissions. However, the trajectory optimization is not treated in this thesis due to its high complexity.

This thesis is part of a collaboration with Professor David Saussié and the masters student Ms. Caroline Dietrich who work for a research group focused on flight path optimization. This research group works together with aeronautic companies like Air Canada, Thales and Roy Avionics & Aircraft Simulation.

1.1. Context

The emergence of low cost airlines in the last two decades has led to traditional airlines, which have always been one of the main drivers of the commercial development aviation, to focus on reducing its costs and increasing its efficiency as an enterprise. For this reason, companies have been focused on reducing indirect operation costs in recent years. However, airlines are reaching the limit where it is possible to reduce indirect operation costs, and therefore, they have begun to focus on the direct operation costs reduction without affecting the provided services.

One of the most significant direct operation costs for an airline is the expenses on fuel, which can be around 30% of its budget. Because of this, airlines have struggled trying to reduce costs in this field not only pushing aircraft manufacturers to design more fuel-efficient aircraft, but also focused on trying to work on the optimization of their flight trajectories.

Nowadays, these flight trajectories consist of segments limited by security, environmental or procedural restrictions. However, the nowadays aircraft are capable to fly nearly continuous trajectories which would be much more efficient. This is why, the R& D collective of commercial aviation is devoting significant efforts to optimize trajectories considering the restrictions and studying the feasibility of removing some of them. In order to perform this task, the aircraft trajectory needs to be studied. Therefore, it is required to know the aircraft aerodynamic model.

EUROCONTROL, the European Organization for Air Safety and the responsible for developing the Single European Sky, has developed an aircraft performance model called BADA (Base of Aircraft Data) for a wide range of commercial aircraft that fly nowadays. This model is a generic model accompanied by a set of data for each aircraft model in order to particularize the generic model. However, being a generic model, it is not considered to be accurate enough to be used in the modeling and optimization of trajectories.

For this reason, in this thesis the main goal is to determine if it is possible to find a more accurate aerodynamic model for two particular long-haul aircraft by using the DATCOM software developed by the US Air Force. To determine the validity of this model, the consumption data of one of them will be compared with BADA model and the model developed by Ms. Dietrich.

1.2. Study considerations

This thesis takes various considerations due the strong conservation of the intellectual property of large companies in the aviation sector. Other considerations have also been conducted to simplify the trajectory modeling problem since it is a really complex problem that requires a large research capacity. The following considerations are also areas in which future research could be focused:

- The studied aircraft will be Boeing 767-300ER and Boeing 777-300ER, two of the most common long-haul aircraft. They have been chosen since they are the two main Air Canada long-haul aircraft.
- Only the cruise phase is considered. This has been decided because it is the flight phase where there are fewer restrictions due to congestion and safety and therefore, where the optimization would provide the maximal profit margin.
- The aircraft will be considered as a mass point body during trajectory modeling. This means that neither the moments caused by aerodynamic forces nor caused by the aircraft deflection surfaces will be considered since during the cruise phase they can be considered negligible in a first order approximation.
- The wind acting where the aircraft is situated will be considered in terms of relative speeds. However, it is not considered the acceleration that it may have at the same point and the force that it would cause on the aircraft. This consideration has been taken since it is negligible and the available data gives only the average wind data over a period of 6 hours.

1.3. Thesis plan

This thesis is divided into seven chapters and four annexes:

- Chapter 1: Contains the thesis introduction. It comprises the context, a list of considerations and the thesis plan.
- Chapter 2: The aerodynamic model designed with DATCOM as well as the operation of the software and a brief analysis of the aerodynamic model results are presented.
- Chapter 3: The BADA aerodynamic model is presented and the method to interpret the data provided by EUROCONTROL is streamlined.
- Chapter 4: The aerodynamic model developed by Ms. Dietrich and its formulation are presented.

- Chapter 5: The trajectory modeling process and several considerations related to it are presented in this chapter.
- Chapter 6: The results of the different simulations are presented and briefly discussed.
- Chapter 7: This chapter presents a thesis summary divided in chapters with its related partial conclusions, the final conclusions and some areas where further research could focus.
- Annex A: This annex shows the DATCOM code used to obtain the data from both aircraft.
- Annex B: This annex shows the Simulink program used to model the aircraft trajectory and its description.
- Annex C: This annex shows the process to obtain an harmonized wind database.
- Annex D: This annex shows how the lift and drag coefficients are deduced in DATCOM simulations.

CHAPTER 2. DATCOOM AERODYNAMIC MODEL

The DATCOM®aerodynamic model will be presented in this chapter. First a introduction to DATCOM software will be given which will help to understand the aircraft geometry modeling process. Finally the aerodynamic results are briefly presented but will be discussed later on.

2.1. Introduction to DATCOM

2.1.1. USAF Stability and Control DATCOM

The United States Air Force Stability and Control DATCOM is a collection, correlation, codification, and recording of best knowledge, opinion, and judgment in the area of aerodynamic stability and control prediction methods. It presents substantiated techniques for use early in the design or concept study phase, to evaluate changes resulting from proposed engineering fixes, and as training on cross training aid. It bridges the gap between theory and practice by including a combination of pertinent discussion and proven practical methods. For any given configuration and flight condition, a complete set of stability and control derivatives can be determined without resort to outside information.

A spectrum of methods is presented [USAF, 1972], ranging from very simple and easily applied techniques to quite accurate and thorough procedures. Comparatively simple methods are presented in complete form, while the more complex methods are often handled by reference to separate treatments. Tables which compare calculated results with test data provide indications of method accuracy. Extensive references to related material are also included.

The report was compiled from September 1975 to September 1977 by the McDonnell Douglas Corporation in conjunction with the engineers at the Flight Dynamics Laboratory at Wright-Patterson Air Force Base.

2.1.2. USAF Stability and Control Digital DATCOM

The United States Air Force Stability and Control Digital DATCOM [Williams and Vukelich, 1979] is a computer program that implements the methods contained in the USAF Stability and Control DATCOM to calculate the static stability, control and dynamic derivative characteristics of fixed-wing aircraft. Digital DATCOM requires an input file containing a geometric description of an aircraft, and outputs its corresponding dimensionless stability derivatives according to the specified flight conditions. The values obtained can be used to calculate meaningful aspects of flight dynamics.

2.1.2.1. *DATCOM Operation*

To be able to work with DATCOM the first requirement is giving the aircraft geometry to the program. This data must be given by standard input parameters. After compiling the file, DATCOM will provide a set of outputs in function of the flight conditions.

Program inputs

Section 3 of the USAF Digital DATCOM Manual Volume I [Williams and Vukelich, 1979] defines the inputs available for modeling an aircraft. These inputs are categorized by namelists to facilitate reading the file in FORTRAN.

- Flight conditions and options: The FLTCON Namelist describes the flight conditions for the case.
- Synthesis parameters: The SYNTHS Namelist allows the user to define the positions of the center of gravity and apexes of the wings.
- Body parameters: The BODY Namelist defines the shape of the body.
- Wing, Horizontal and Vertical Tail parameters: The WGPLNF, HTPLNF and VTPLNF Namelists define the wing, horizontal tail and vertical tail, respectively.
- High Lift and Control Devices: Using the SYMFLP and ASYFLP Namelists, flaps, elevators, and ailerons can be defined.
- Other Inputs: Other Digital DATCOM inputs include power effects (propeller and jet), ground effects, trim tabs, and experimental data.

Program outputs

For each configuration, the outputs provided by DATCOM will be the stability coefficients and derivatives for each angle of attack specified. Next lists specifies the basic outputs.

- C_L: Lift coefficient.
- C_D: Drag coefficient.
- C_l: Pitching moment coefficient.
- C_n: Normal force coefficient.
- C_A: Axial force coefficient.
- $C_{L\alpha}$: Lift curve slope (derivative of lift coefficient with respect to angle of attack).
- $C_{m\alpha}$: Pitching moment curve slope (derivative of pitching moment coefficient with respect to angle of attack.
- $C_{y\beta}$: Derivative of side-force coefficient with respect to side-slip angle.
- $C_{n\beta}$: Derivative of yawing-moment coefficient with respect to side-slip angle.
- $C_{l\beta}$: Derivative of rolling-moment coefficient with respect to side-slip angle.

2.1.2.2. DATCOM limitations

Inlets, external stores, and other protuberances cannot be input because Digital DATCOM analyzes the fuselage as a body of revolution. This simplification affects the aircraft drag coefficient.

Dynamic derivatives are not output parameters for aircraft that have wings that are not straight-tapered or have leading edge extensions. This problem can be overcome by using experimental data for the wing-body (using non-straight tapered wing).

There is no method to input twin vertical tails mounted on the fuselage, although there is a method for H-Tails. This problem can be addressed by approximating the twin vertical tails as a single equivalent vertical tail mounted to the fuselage.

Digital DATCOM cannot provide outputs for the control derivatives with regard to the rudder control surface. According to the manual, there is no any input parameters that define the rudder geometry.

Digital DATCOM cannot analyze three lifting surfaces at once, such as a canard-wing-horizontal tail configuration. This problem can be addressed by superposition of lifting surfaces through the experimental input option.

2.2. Aircraft geometric modeling process

In this section the input parameters used to get the aerodynamic coefficients are presented. For each namelist, the input parameters are presented in the same order that they are presented in the input file. The code used for this thesis is shown in Appendix A.

Considering the small quantity of data available regarding to aircraft geometry, only the basic geometry of the aircraft has been introduced to DATCOM in this project. This means that only body, wings, horizontal and vertical tails will be defined.

Boeing 767-300ER and Boeing 777-300ER have been modeled since they are the most common long-haul aircraft of Air Canada fleet.

Data has been obtained from 3D graphics from Boeing website. and is shown in Figure 2.1 [Boeing, 1998]. From this drawings all the input parameters have been approximated. With this drawings it is impossible to determine the aircraft airfoil section and no data has been found in the network since Boeing keep it as critical private data. Therefore, an arbitrary NACA profile will be used for the airfoil section (See Section 2.2.6.).

2.2.1. Flight conditions (FLTCON)

This namelist allows the program to compute the different coefficients in different cases, and by interpolation other desired results may be obtained.

- Mach number (M): Mach will be studied from Mach 0.1 to the aircraft's maximum cruise speed which is Mach 0.86. Since the modeling is carried out with subsonic flow, the bigger the Mach number is, the less reliable the results are.

- Altitude (h): A wide range of altitudes has been considered from surface to FL440.

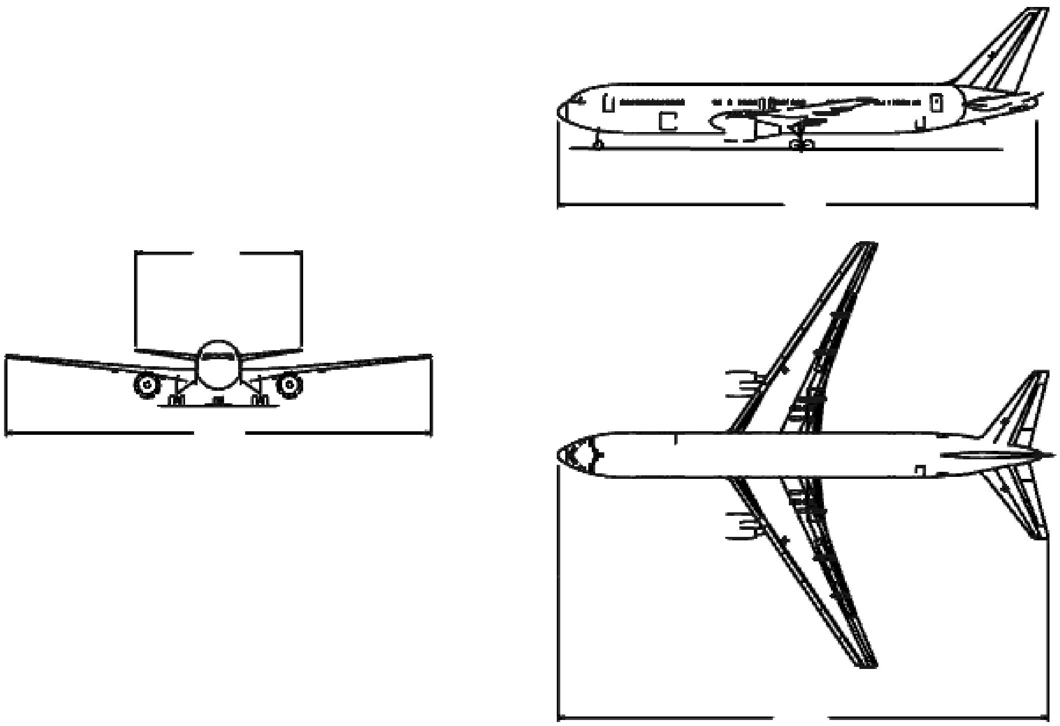


Figure 2.1: Boeing 767-300 Drawing[Boeing, 1998]

- Angle of attack (α): A wide range of angles has been considered from -16 to 24.
- Flight path angle (γ): Since the main goal is to study the cruise phase, γ has been set to $0\,\mathrm{deg}$

2.2.2. Synthesis parameters

This namelist provides the different parameters regarding to other namelists.

- Center of gravity: Location of the center of gravity. This value might change depending on the flight conditions so an arbitrary value within the normal values has been chosen trying to make the aircraft as stable as possible regarding the pitch moment.
- Wing apex: Location of theoretical wing apex.
- Horizontal tail apex: Location of theoretical horizontal tail apex.
- Vertical tail apex: Location of theoretical vertical tail apex.

2.2.3. Body parameters

This namelist contains the parameters regarding to the fuselage (See Figure 2.2). Fuselage is considered symmetrical in the y direction but not in the z direction since it is not a perfect cylinder.

- X (vector): Longitudinal distance measured (at this distance, X_i, the following parameters are measured).
- S (vector): Cross sectional area at X_i.
- ZU (vector): Z coordinates at upper body surface at station X_i.
- ZL (vector): Z coordinate at lower body surface at station X_i.

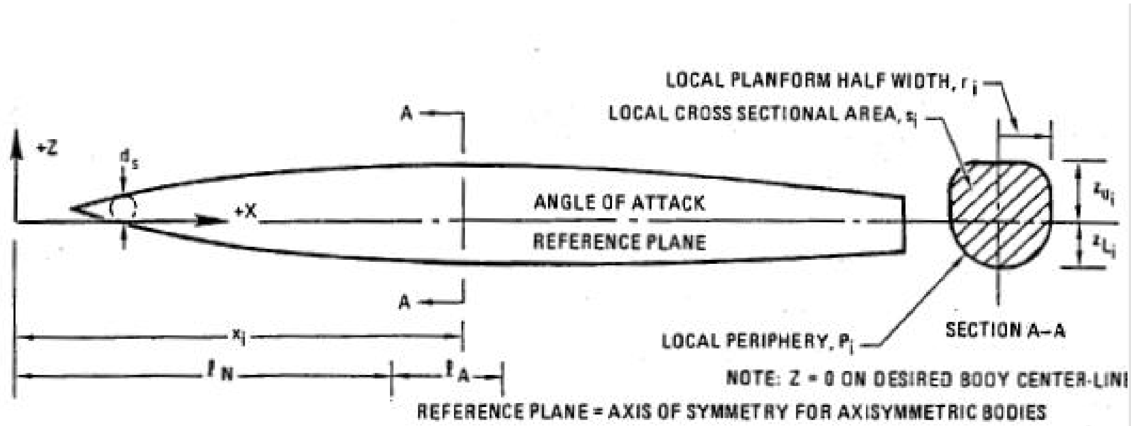


Figure 2.2: Body parameters [Williams and Vukelich, 1979]

2.2.4. Wing characteristics

This namelists contains the different parameters regarding to the wing (See Figure 2.3). Cross sectional parameters are defined later on thanks to the NACA controls (See Section 2.2.6.).

- CHRDTP (c_t): Tip chord.
- CHRDDBP (c_b): Chord at break point.
- CHRDDR (c_r): Root chord.
- SSPN($b/2$): Semi-span theoretical panel from theoretical root chord.
- SSPNOP ($b_o^*/2$): Semi-span outboard panel.
- SSPNE ($b^*/2$): Semi-span theoretical panel from theoretical root chord.
- CHSTAT (x/c): Reference chord station from inboard and outboard panel sweep angles, fraction of chord.
- TWISTA (θ): Twist angle, negative leading edge rotated down (from exposed root to tip).
- SAVSO ($\Lambda_{X/C}$): Outboard panel sweep angle.
- DHADO (Γ_0): Dihedral angle of outboard panel.
- TYPE: Straight tapered planform.

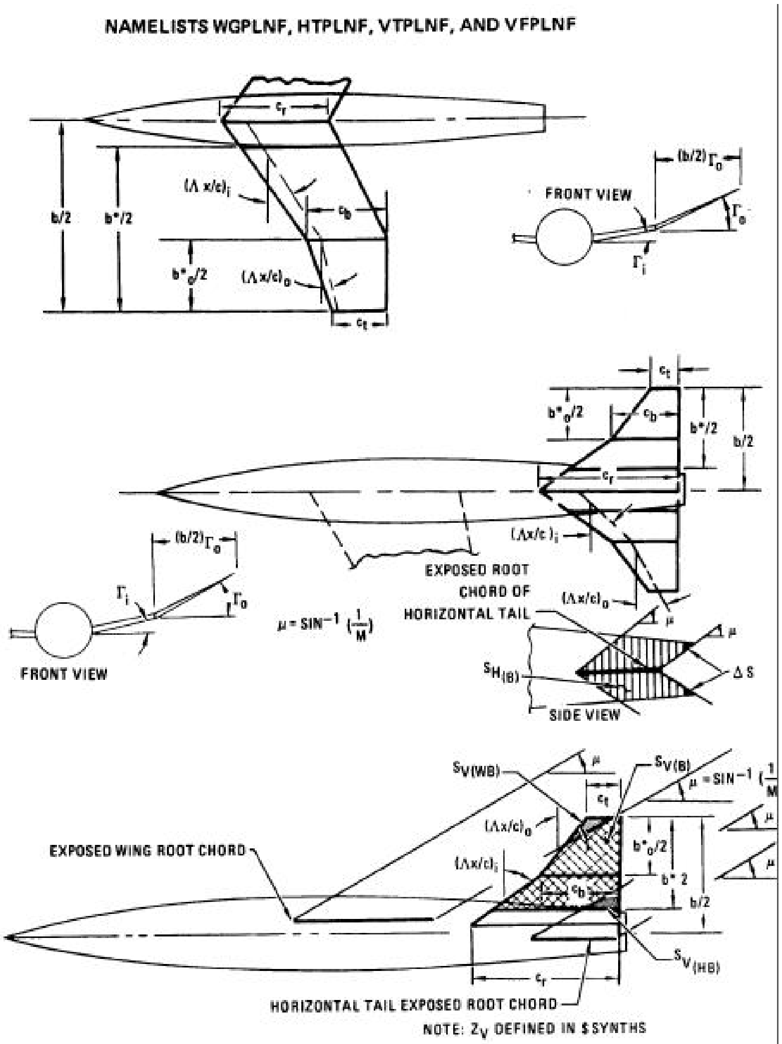


Figure 2.3: Wing parameters [Williams and Vukelich, 1979]

2.2.5. Horizontal and vertical tail characteristics

This namelist contains the different parameters regarding to the horizontal and vertical tail (See Figure 2.3). Cross sectional parameters are defined later on thanks to the NACA controls (See Section 2.2.6.).

- CHRDTP (c_t): Tip chord.
- CHRDDBP (c_b): Chord at break point.
- SSPN($b/2$): Semi-span theoretical panel from theoretical root chord.
- SSPNE ($b^*/2$): Semi-span theoretical panel from theoretical root chord.
- CHSTAT (x/c): Reference chord station from inboard and outboard panel sweep angles, fraction of chord.
- TWISTA (θ): Twist angle, negative leading edge rotated down (from exposed root to tip).

- DHADI (Γ_0): Dihedral angle of outboard panel.
- TYPE: Straight tapered planform.

2.2.6. NACA controls

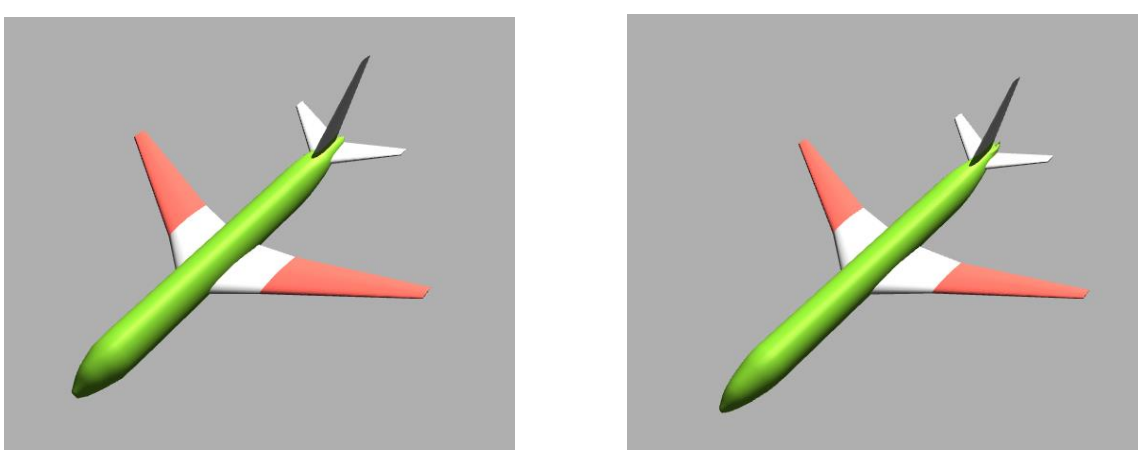
The NACA airfoils are airfoil shapes for aircraft wings developed by the National Advisory Committee for Aeronautics (NACA). The shape of the NACA airfoils is described using a series of digits following the word ”NACA”. The parameters in the numerical code can be entered into equations to precisely generate the cross-section of the airfoil and calculate its properties. NACA controls have been defined for wing, horizontal tail and vertical tail. Since this does not use to be public data from the manufacturer different simulations have been carried out and the solution that gives more reasonable values of lift coefficient and drag coefficient has been chosen.

2.3. Results and analysis

Depending on the input data, DATCOM provides a different number of outputs and an aircraft sketch. In this section, the results obtained with DATCOM will be presented and analyzed briefly. First of all, the aircraft sketch will be presented. Then, the aerodynamic coefficients will be compared between the two aircraft and an A-320. Finally, the aerodynamic coefficients variation with altitude and speed will be studied.

2.3.1. Aircraft Sketches

Figures 2.4(a) and 2.4(b) show the sketches provided by DATCOM. It is possible to realize that the vertical tail looks too big, but DATCOM says that these sketches are just sketches and not accurate drawings. Therefore, it is possible to see that both aircraft do not present strange forms.



(a) Boeing 767-300ER

(b) Boeing 777-300ER

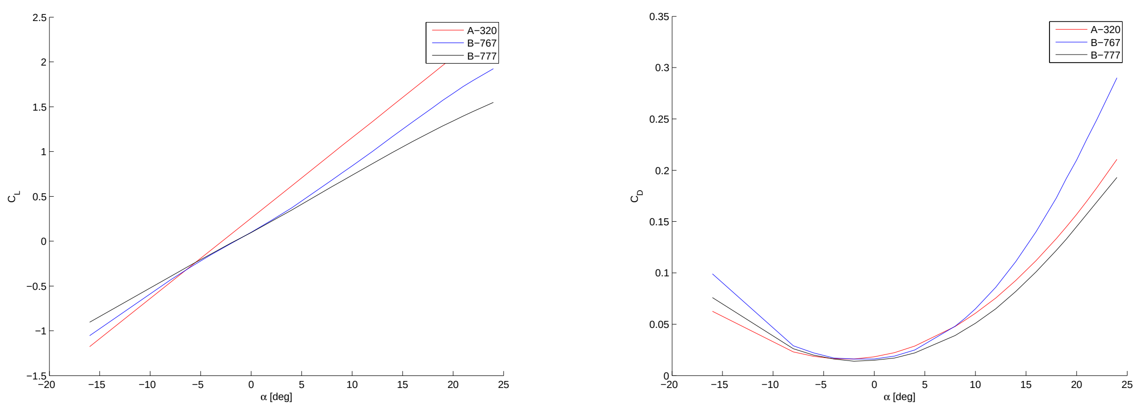
Figure 2.4: Aircraft sketches

2.3.2. Comparison between different aircraft

In this section, lift and drag coefficients function of the angle of attack have been compared. They have been compared in function of the angle of attack since it is the most important factor that affects these coefficients. Some data was available for the A-320 and it has been used to compute the same coefficients.

Figure 2.5(a) and 2.5(b) show a comparison between the two aircraft designed in this project using DATCOM, the Boeing 767-300ER and 777-300ER, and the A-320. In these figures it is possible to see that the Airbus has slightly higher lift coefficient values than the others. However, drag coefficient values, which is the most important for this project, are really similar for the angle of attack values of normal operation.

Although the lift coefficient looks like a line, the only values considered are inside a normal range of operation, which is approximately between -5 and 15 degrees. For higher values the boundary layer is detached and then these values drop significantly. The values outside this range should not be considered reliable.



(a) Lift coefficient

(b) Drag coefficient

Figure 2.5: Aircraft aerodynamic coefficients comparison

2.3.3. Variation with altitude and speed

In this section, the aerodynamic coefficients variation in function of altitude and Mach number has been studied. These comparisons have been performed using the Boeing 767-300ER.

The lift coefficient variation function of altitude is almost zero. For the same angle of attack and Mach number the lift coefficient varies less than 0.01% in a range of altitudes of 44.000 ft. However, in Figure 2.6, it is possible to see that the drag coefficient slightly increases with altitude.

The lift coefficient variation function of Mach number (Figure 2.7(a)) becomes more important as the Mach number increases. This variation is a reduction of lift coefficient as Mach number increases. The same situation occurs on the drag coefficient (Figure 2.7(b)), probably because they have a directly proportional relation.

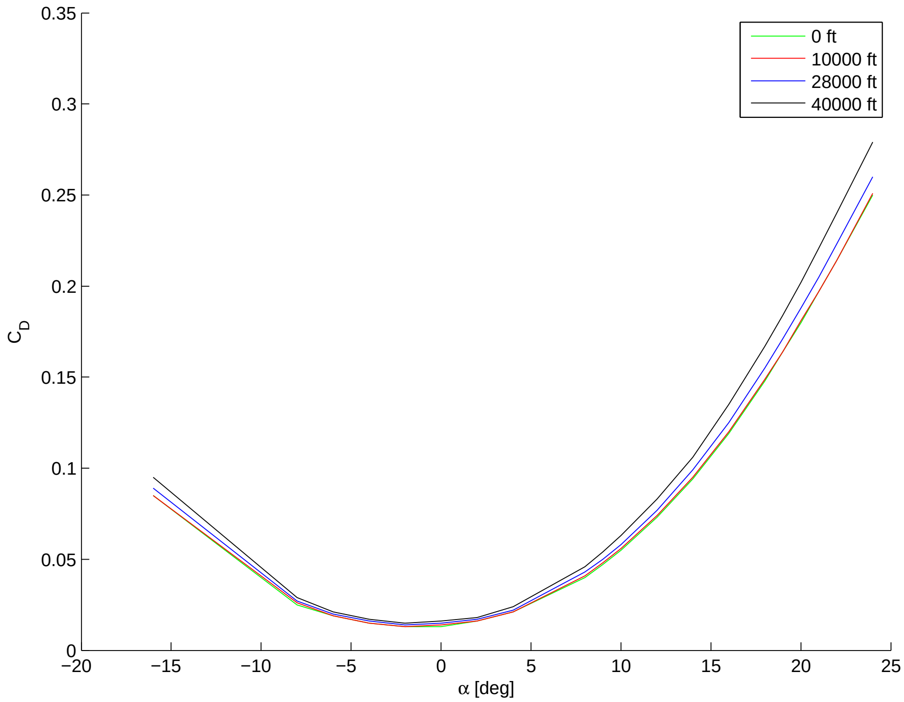
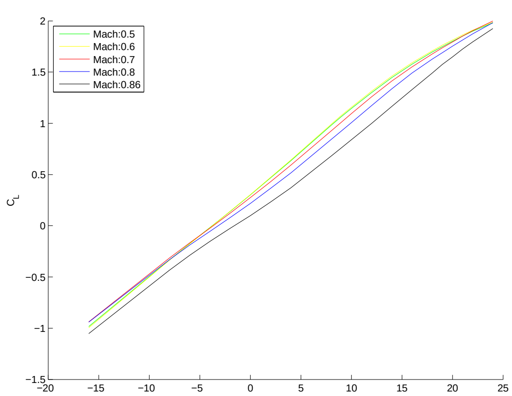
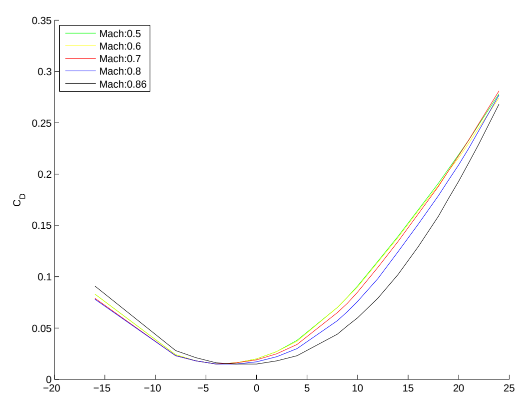


Figure 2.6: Drag coefficient altitude comparison



(a) Lift coefficient



(b) Drag coefficient

Figure 2.7: Aerodynamic coefficients variation function of Mach number

CHAPTER 3. BADA AERODYNAMIC MODEL

In this chapter, the BADA aerodynamic Model will be presented. First, the BADA Performance model will be presented. Then, a description of how to deal with BADA files used in this project will be carried out. Finally, the BADA aerodynamic model applied to this project will be developed.

3.1. Introduction to BADA

Base of Aircraft Data (BADA), is an Aircraft Performance Model (APM) developed and maintained by EUROCONTROL through active cooperation with aircraft manufacturers and operating airlines [Nuic, 2010]. The information and data contained in BADA is designed for:

- trajectory simulation in the air traffic modeling and simulation tools which are used to support R&D, validation and assessment of new ATM concepts, ATC procedures, advanced controller decision support tools and equipment before they are introduced into operational service.
- trajectory prediction in the ground based operational ATM systems (Flight Data Processing Systems) to better plan traffic flows, reduce delays and operating costs and minimize adverse environmental impact.
- environmental studies in terms of aircraft emissions assessments.

This project covers mainly the first use, but it could also be related with the third use if necessary.

BADA is made out of two components:

- Model specifications: theoretical fundamentals provided in form of generic polynomial expressions used to calculate aircraft performance parameters.
- Data-sets: a data-set for a given aircraft contains the specific value of the coefficients present in the model specification that particularize the BADA model for a specific aircraft type.

There are two families of BADA Aircraft Performance Model based on the same modeling approach and components:

- BADA Family 3: today's standard for aircraft performance modeling providing a 90% coverage of the current aircraft types operating in the ECAC airspace. The primary objective of BADA 3 is to model aircraft performance over the normal operations part of the flight envelope and to meet today's requirements for aircraft performance modeling and simulation.
- BADA Family 4: a newly developed model intended to meet advanced functional and precision requirements of the new ATM systems and R&D providing a 60%

coverage of the current aircraft types operating in ECAC airspace. BADA 4 provides accurate modeling of aircraft over the entire flight envelope and enables modeling and simulation of advanced concepts of future systems.

Since BADA 4 family is still not available for the R&D community, BADA 3 family has been used in this project.

3.2. BADA aerodynamic model

The Total-Energy Model that BADA uses [Nuic, 2010], equates the rate of work done by forces acting on the aircraft to the rate of increase in potential and kinetic energy, that is:

$$(T-D)\cdot V_{TAS} = m\cdot g\cdot \frac{dh}{dt} + m\cdot V_{TAS}\cdot \frac{dV_{TAS}}{dt} \tag{3.1}$$

where T is the thrust, D the aerodynamic drag, V_{TAS} the true airspeed (TAS), m the aircraft mass, and dh/dt rate of climb. In this part of the project, the main goal is to obtain the C_D, but in order to solve this, BADA first need to know the C_L. Assuming that the flight path angle and bank angle are zero, C_L may be obtained using Equation 3.2.

$$C_L = \frac{2\cdot m\cdot g}{\rho\cdot {V_{TAS}}^2\cdot S} \tag{3.2}$$

Under nominal conditions, the drag coefficient C_D is specified as a function of the lift coefficient C_L (Equation 3.3).

$$C_D = C_{D0,CR} + C_{D2,CR}\cdot {C_L}^2 \tag{3.3}$$

$C_{D0,CR}$ and $C_{D2,CR}$ are two constant values depending on the aircraft that are given by BADA. Next section show how to extract these values from the .OPF files. The aerodynamic drag force is then determined from the drag coefficient in the standard manner (Equation 3.4).

$$D = \frac{\rho\cdot {V_{TAS}}^2\cdot S\cdot C_D}{2} \tag{3.4}$$

3.3. Understanding BADA files

All data provided by BADA Revision 3.10 are organized into six types of files [Nuic, 2010]:

- three Synonym Files,
- a set of Operation Performance Files.
- a set of Airline Procedure Files.
- a set of Performance Table Files.
- a set of Performance Table Data.
- a Global Parameter File.

For this project, only the Operation Performance File (.OPF) has been used. This file provides for each aircraft type which is directly supported the parameter values for the mass, flight envelope, drag, engine thrust and fuel consumption. An OPF file is shown in Listing 3.1.

Listing 3.1: .OPF File

```
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC B763__ OPF CCCCCCCCCCCCCC/
CC                                                                    /
CC                  AIRCRAFT PERFORMANCE                              /
CC                  operational    files                              /
CC                                                                    /
CC      BADA RCS File Id                                              /
CC     File Name         Current Revision      Last Modification      /
CC                       revision     date      revision      date    /
CC     B763__.OPF          3.3    2000/12/06    3.1.1.1    2000/08/01 /
CC                                                                    /
CC    BADA Revision:                                                  /
CD    Rev  3.3                                                        /
CC====== Actype ======================================================/
CD    B763__         2 engines     Jet                        H       /
CC          B767-300ER with PW4060 engines                   wake     /
CC                source = Aeromaritime Manuel d'Exploitation         /
CC====== Mass (t) ====================================================/
CC     reference      minimum      maximum     max payload  mass grad /
CD      .15000E+03   .89900E+02   .18140E+03   .37503E+02   .13000E+00 /
CC====== Flight envelope =============================================/
CC      VMO(KCAS)       MMO        Max.Alt       Hmax       temp grad /
CD      .35000E+03   .86000E+00   .43000E+05   .33800E+05  -.11000E+03 /
CC====== Aerodynamics ================================================/
CC Wing Area and Buffet coefficients (SIM)                            /
CCndrst  Surf(m2)     Clbo(M=0)      k           CM16                 /
CD 5    .28330E+03   .00000E+00   .00000E+00   .00000E+00             /
CC    Configuration characteristics                                   /
CC n Phase  Name     Vstall(KCAS)    CD0          CD2        unused   /
CD 1 CR   Clean      .16500E+03   .14000E-01   .49000E-01   .00000E+00 /
CD 2 IC   Flap15     .12200E+03   .00000E+00   .00000E+00   .00000E+00 /
CD 3 TO   Flap15     .12200E+03   .00000E+00   .00000E+00   .00000E+00 /
CD 4 AP   Flap15     .12200E+03   .35000E-01   .45000E-01   .00000E+00 /
CD 5 LD   Flap30     .11300E+03   .57000E-01   .40000E-01   .00000E+00 /
CC    Spoiler                                                         /
CD 1       RET                                                        /
CD 2       EXT                              .00000E+00   .00000E+00 /
CC    Gear                                                            /
CD 1       UP                                                         /
CD 2       DOWN                .18000E-01   .00000E+00   .00000E+00 /
CC    Brakes                                                          /
CD 1       OFF                                                        /
CD 2       ON                               .00000E+00   .00000E+00 /
CC====== Engine Thrust ===============================================/
CC          Max climb thrust coefficients (SIM)                       /
CD      .29600E+06   .50900E+05   .41900E-10   .56600E+01   .55000E-02 /
CC       Desc(low)    Desc(high)    Desc level   Desc(app)    Desc(ld)  /
CD      .13000E-01  -.74000E-01   .10000E+05   .14000E+00   .28000E+00 /
CC       Desc CAS     Desc Mach     unused       unused       unused    /
CD      .29000E+03   .78000E+00   .00000E+00   .00000E+00   .00000E+00 /
CC====== Fuel Consumption ============================================/
```

```
CC   Thrust Specific Fuel Consumption Coefficients                      /
CD     .76300E+00   .14300E+04                                          /
CC   Descent Fuel Flow Coefficients                                     /
CD     .18800E+02   .69400E+05                                          /
CC   Cruise Corr.    unused       unused       unused       unused    /
CD     .10347E+01   .00000E+00   .00000E+00   .00000E+00   .00000E+00 /
CC====== Ground ====================================================/
CC        TOL          LDL        span         length       unused    /
CD     .29250E+04   .17400E+04   .47600E+02   .54900E+02   .00000E+00 /
CC==================================================================/
```

For this section, the sought values $C_{D0,CR}$ and $C_{D2,CR}$, may be found in the *Aerodynamics* set, *Configurations characteristics* subset, *Clean* row, last two columns. The file from Listing 3.1 is the public version for an .OPF file although coincidentally it is for the same aircraft than the one treated in this thesis. However, this paper has been able to obtain the values for a more recent version of BADA 3 family and next values have been used.

$$C_{D0,CR} = .18000E-01 \text{ and } C_{D2,CR} = .48000E-01$$

CHAPTER 4. DIETRICH AERODYNAMIC MODEL

The Dietrich aerodynamic model will be presented in this chapter. First of all, the mainspring of this model will be explained. Then, the drag formulation will be given. Finally, a summary will be presented to clarify the formulation.

4.1. Introduction

The drag force [Dietrich, 2014] is the main responsible for the aircraft fuel consumption. Since the BADA 3 model presents a really simple drag model, it may not be considered really accurate for a wide range of different flight envelopes. Because of this, a more accurate model has been developed by Ms. Dietrich through research on the existing literature and simulation of different models. Since this research was carried out in order to deal with fuel consumption issues, the lift force will be formulated in the same way that BADA 3 model (Equation 3.2). In this chapter, the drag coefficient, C_D, will be formulated instead of the aerodynamic drag force but they can be related with the Equation 5.9.

4.2. Drag Model

Aerodynamic drag [Anderson, 2005] is the fluid drag force that acts on any moving solid body in the direction of the fluid free-stream flow. From the flow field perspective drag is generally divided in three main sources: induced drag, (C_{D_i}), parasitic or skin friction drag, (C_{D_f}) and wave drag, (C_{D_w}). Other sources are also proved to exist but the significance in comparison to the others is usually negligible.

Then the C_D can be expressed as:

$$C_D = C_{D_i} + C_{D_f} + C_{D_w} \tag{4.1}$$

4.2.1. Induced Drag

The induced drag is the main drag component and it varies with the square of the lift coefficient [Dietrich, 2014]. This induced drag is caused because of vortex created on the wingtips. The induced drag coefficient, C_{D_i}, is approximated as:

$$C_{D_i} = \frac{{C_L}^2}{e \cdot \pi \cdot \lambda} \tag{4.2}$$

$$\lambda = \frac{b^2}{S} \tag{4.3}$$

where λ is the Aspect Ratio (Equation 4.3), b and S are the wing span and wing surface respectively and e is the Oswald's coefficient. The Oswald's coefficient e, depends on the wing span lift distribution and it is generally between 0,75 and 0,85 for a normal wing. According to Dietrich, this value e must be corrected with Jobe's expression of e' to consider also the aircraft's body [Jobe, 1984].

$$e = e' \cdot (1 - (d/b)^2) \tag{4.4}$$

where d is the body diameter and Dietrich uses the Hull's estimation of e' [Hull, 2007] as:

$$e' = (1 - 0.045 \cdot \lambda^{0.068}) \cdot (1 - 0.0227 \cdot {\Lambda_{qc}}^{1.615}) \tag{4.5}$$

where Λ_{qc} is the quarter-chord sweep angle. So to sum up, the C_{D_i} can be modeled using Equations 4.2 to 4.5 in function of C_L, d, b, λ and Λ_{qc}.

$$C_{D_i} = \frac{{C_L}^2}{(1 - 0.045 \cdot \lambda^{0.068}) \cdot (1 - 0.0227 \cdot {\Lambda_{qc}}^{1.615}) \cdot (1 - (d/b)^2) \cdot \pi \cdot \lambda} \tag{4.6}$$

4.2.2. Parasitic Drag

The parasitic drag is divided in three terms: form drag, friction drag and interference drag. Usually, form and friction drag are regrouped inside the same coefficient C_{D_f}. Then, interference drag, $C_{D_{interference}}$, will be expressed independently. Dietrich finally chooses the Filippone and Gur [Filippone, 2008] estimation of the skin friction drag as:

$$C_{D_{parasitic}} = C_{D_f} + C_{D_{interference}} \tag{4.7}$$

a Profile Drag

Profile drag is the sum of form drag and skin friction drag. The form drag depends on the wing form, size and longitudinal section. It is caused by the airflow around the aircraft surfaces and the boundary layer detachment due to the viscous effects. Skin friction drag is caused by the fluid friction with the surface. It depends on the body wet surface, S_{wet}, which is the total surface in contact with the fluid. Dietrich uses the expression of Filippone and Gur [Filippone, 2008] for the profile drag, (C_{D_f}), that estimates that C_{D_f} is the sum of all the k elements of the aircraft:

$$C_{D_f} = \sum C_{f_k} \cdot FF_k \cdot \frac{S_{wet_k}}{S_{ref_k}} \tag{4.8}$$

where C_{f_k} is the flat plate friction coefficient, FF_k is the form factor, S_{wet_k} is the wet surface and S_{ref_k} is the reference surface. The Dietrich model uses the next Hull's expression [Hull, 2007] to obtain the flat plate friction coefficient for aircraft modeling:

$$C_{f_k} = \frac{0,455}{(\log_{10} Re_k)^{2,58}} \tag{4.9}$$

Re_k is the Reynolds number and it depends on the airflow speed V, characteristic length l and flow viscosity:

$$Re_k = \frac{V \cdot l}{\nu} \tag{4.10}$$

Regarding to the FF_k Hull expresses it as [Hull, 2007]:

$$\begin{aligned} FF_W &= 1 + 1,6 \cdot (t/c)_W + 100 \cdot {(t/c)^4}_W \\ FF_B &= 1 + \frac{60}{{(l/d)^3}_B} + 0,0025 \cdot (l/d)_B \end{aligned} \tag{4.11}$$

where l is the body length, d the body diameter and $(t/c)_W$ the thickness/chord ratio at the wing root. Other models are also given to compute other aircraft elements as the stabilizers or the nose but they will not be considered for this project. To sum up, the profile drag coefficient, C_{D_f}, could be reformulated using Equations 4.8 to 4.11 as:

$$\begin{aligned} C_{D_{f_W}} &= \frac{0,455}{(\log_{10}\frac{V\cdot mac}{\nu})^{2,58}} \cdot \left(1+1,6\cdot(t/c)_W + 100\cdot{(t/c)^4}_W\right)\cdot\frac{S_{wet_W}}{S_{ref_W}} \\ C_{D_{f_B}} &= \frac{0,455}{(\log_{10}\frac{V\cdot l}{\nu})^{2,58}} \cdot \left(1+\frac{60}{{(l/d)^3}_B} + 0,0025\cdot(l/d)_B\right)\frac{S_{wet_B}}{S_{ref_B}} \\ C_{D_f} &= C_{D_{f_W}} + C_{D_{f_B}} \end{aligned} \tag{4.12}$$

b Interference Drag

The interference drag is caused by the flux changes due to the presence of other elements. The Dietrich model uses the Tétrault expression [Tétrault et al., 2000] obtained through CFD in function of the aircraft sweep angle, ϕ, the thickness/chord wing root ratio and the critical Reynolds, Re_c.

$$\begin{aligned} C_{D_{interference}} = 0,1112 - 0,2572\cdot\sin\phi + 3,440\cdot(t/c) - 0,02097\cdot\log_{10}Re_c \\ + 0,09009\cdot\sin^2\phi - 2,549\cdot(t/c)\cdot\sin\phi + 0,03010\cdot\log_{10}Re_c\cdot\sin\phi \\ - 0,1462\cdot(t/c)\cdot\log_{10}Re_c \end{aligned} \tag{4.13}$$

c Summary

To sum up, the set of Equations 4.14 shows how the parasitic drag coefficient is obtained by also using Equations 4.7, 4.12 and 4.13.

$$\begin{aligned} C_{D_{parasitic}} &= C_{D_f} + C_{D_{interference}} \\ C_{D_f} &= C_{D_{f_W}} + C_{D_{f_B}} \quad 4.12 \\ C_{D_{interference}} &= f(Re_c, \phi, (t/c)) \quad 4.13 \end{aligned} \tag{4.14}$$

4.2.3. Wave Drag

The wave drag, C_{D_w}, appears at subsonic speeds, when it is locally accelerated to a supersonic speed and then, because of the shock wave, it goes back to subsonic speed. This form of drag becomes more important when the speeds become closer to Mach=1. The Dietrich model uses the Lock's law expression to compute the C_{D_w}:

$$C_{D_w} = \begin{cases} 0 & M \leq M_{cr} \\ 20\cdot(M-M_{cr})^4 & M > M_{cr} \end{cases} \tag{4.15}$$

where M_{cr} is the critical Mach, (i.e. the Mach number when the airflow becomes supersonic in one point of the aircraft). The M_{cr} can be expressed as function of the aircraft's geometrical data.

$$M_{cr} = \frac{\kappa_a}{\cos\Lambda_{hc}} - \frac{C_L}{10\cos^2\Lambda_{hc}} - \frac{t/c}{\cos^3\Lambda_{hc}} - \sqrt[3]{\frac{0,1}{80}} \tag{4.16}$$

where κ_A is the Korn's factor and the Dietrich model set this value as 0,95. As reminder, Λ_{hc} is the half-chord sweep angle.

4.3. Summary

After developing the different drag coefficients, it is possible to summarize all the equations seen in this chapter to have a global idea of the drag. Returning to Equation 4.1:

$$C_D = C_{D_i} + C_{D_f} + C_{D_w}$$

and also using Equations 4.6, 4.14 and 4.15 it is possible to obtain that the C_D depends on several different magnitudes.

$$C_D = f(C_L, \lambda, \Lambda_{qc}, d, b, V, mac, \nu, (t/c)_w, S_{wet_W}, S_{ref_W}, S_{wet_B}, S_{ref_B}, l, \phi, \kappa_A, \Lambda_{hc}) \tag{4.17}$$

This equation clearly shows the complexity of modeling an aerodynamic force applied to an aircraft. However, with the nowadays computers, there are new techniques that estimate these forces using numerical methods in order to solve fluid flow problems. They are provided of the exact aircraft continuous geometry and not only from some discrete parameters as in this case.

CHAPTER 5. TRAJECTORY MODELING

In this chapter the trajectory modeling process is presented. First of all, the equations that will drive the simulation are presented. Then, the atmospheric and consumption models related with these equations will be presented. Finally, different aspects also related to the trajectory modeling as the coordinates conversion, the wind modeling or the flight path computing will be presented.

5.1. 3-D point-mass aircraft model

The 3D point-mass aircraft model with horizontal directions x and y and vertical altitude h is represented as follows [Howe-Veenstra, 2011]:

$$
\begin{aligned}
m \cdot \dot{V} &= T - D - m \cdot g \cdot \sin\gamma - m \cdot \dot{W}_V && (5.1)\\
m \cdot V \cdot \cos\gamma \cdot \dot{\psi} &= L \cdot \sin\phi - m \cdot \dot{W}_\psi && (5.2)\\
m \cdot V \cdot \dot{\gamma} &= L \cdot \cos\phi - m \cdot g \cdot \cos\gamma + m \cdot \dot{W}_\gamma && (5.3)\\
\dot{x} &= V \cdot \cos\gamma \cdot \sin\psi + W_x && (5.4)\\
\dot{y} &= V \cdot \cos\gamma \cdot \cos\psi + W_y && (5.5)\\
\dot{h} &= V \cdot \sin\gamma + W_h && (5.6)
\end{aligned}
$$

These six equations represent the dynamic equations of motion governing the rate of change in velocity, V, heading angle, ψ, flight path angle, γ, East coordinate, x, North coordinate, y, and altitude, h respectively. In addition to the above core dynamics, the mass is not considered constant. Therefore, a mass variation assessment needs to be carried out. This expression and all the concepts regarding to mass variation will be explained in Section 5.3.

Since the aircraft is not flying at constant airspeed but at constant Mach, an expression for the Mach number must be given (Equation 5.7).

$$M = \frac{V}{a} \tag{5.7}$$

where a is the speed of sound which depends on temperature, and therefore, on the altitude. To obtain a, the International Standard Atmosphere will be assumed (Section 5.2.).

Additional definitions of lift, L, and drag, D, are also required to complete the model.

$$
\begin{aligned}
L &= \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot C_L && (5.8)\\
D &= \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot C_D && (5.9)
\end{aligned}
$$

These equations rely on both the atmospheric model (Section 5.2.) and the aerodynamic coefficients.

Wind components can be described as the components along velocity, heading and flight-path directions. However they are more commonly described in terms of their East, North and Vertical components. The wind model will be explained in Section 5.5.

5.2. Atmosphere Model

As seen in Section 5.1., some aspects as Mach expression, the expressions for the aerodynamic forces, or the atmospheric parameters are required to study aircraft trajectories. The important parameters for this particular formulation a and ρ.

The International Standard Atmosphere model (ISA) will be used for this thesis since it is the main model used in the aeronautic world. The International Standard Atmosphere is an atmospheric model of how the pressure, temperature, density and viscosity of the Earth's atmosphere change over a wide range of altitudes or elevations. It has been established to provide a common reference for temperature and pressure and consists of tables of values at various altitudes, plus some formulas by which those values were derived. Since most commercial flights fly at altitudes less than $h_T \approx 11Km$, it is possible to consider only the troposphere equations.

$$T(h) = T_0 - \alpha \cdot h \tag{5.10}$$

$$P(h) = P_0 \cdot \left(\frac{T(h)}{T_0}\right)^{-\frac{g}{\alpha R}} \tag{5.11}$$

$$\rho(h) = \rho_0 \cdot \left(\frac{T(h)}{T_0}\right)^{-\frac{g}{\alpha R}-1} \tag{5.12}$$

(5.13)

where $T_0 = 288.15\ K$, $\alpha = 6.5 \cdot 10^{-3}\ K/m$ and T, P and ρ are expressed in K, Pa and Kg/m^3 respectively. The speed of sound a is also function of the temperature, which has already been obtained, and is computed using Equation 5.14.

$$a = \sqrt{\gamma \cdot R \cdot T} \tag{5.14}$$

γ is the adiabatic index, which is widely assumed as 1.4 and R is the molar gas constant divided by the Boltzmann constant.

5.3. BADA Consumption Model

Since DATCOM does not provide any model for aircraft acceleration and fuel consumption, BADA model will be applied to study the aircraft mass variation. Nuic [Nuic, 2010] expresses the consumption model for jets (including turbofans) and turboprops depending on the flight envelope. Since the the Boeing 767-300ER uses turbofans engines, the jet expression for cruise will be used (Equation 5.15).

$$f_{cr} = \eta \cdot T \cdot C_{fcr} \tag{5.15}$$

where η is the thrust specific fuel consumption in $[kg/(minkN)]$, C_{fcr} is a constant that can be found in the BADA files and f_{cr} is the cruise fuel flow in $[kg/min]$. Since the mass variation, $\dot{m}$, depends only on a known constant C_{fcr}, the thrust T, which is known and the fuel consumption, Equation 5.16 can be added to Equations 5.1 to 5.6 to perform the aircraft trajectory assessment.

$$\dot{m} = \frac{-f_{cr}}{60} \tag{5.16}$$

5.4. Geodesic coordinates

Although Equations 5.1 to 5.6 work mainly with North and East coordinates expressed in length units, the aircraft navigation environment usually works with geodesic coordinates, latitude, longitude and altitude, to determine an unique position in the earth. WGS84 model will be used since it is the standard model used by commercial aviation.

The WGS84 origin is meant to be the Earth's center of mass. The datum surface is an ellipsoid with a major equatorial radius a of 6.378 km and a polar semi-minor axis b of 6.356 km. This system is an aviation standard because is the one tat Global Positioning System (GPS) uses.

For three x, y, z known coordinates, it is possible to determine the latitude, λ, longitude, ϕ, and geodetic altitude, h. The major axis of the ellipsoid a and the minor axis b must be also known [Olmos, 2014].

$$
\begin{aligned}
e &= \sqrt{1-\frac{b^2}{a^2}} \\
e' &= \frac{a\cdot e}{b} \\
f &= 1-\frac{a}{b} \\
r &= \sqrt{x^2+y^2} \\
E^2 &= a^2-b^2 \\
F &= 54\cdot b^2\cdot z^2 \\
G &= r^2+(1-e^2)\cdot z^2-e^2\cdot E^2 \\
c &= \frac{e^4\cdot F\cdot r^2}{G^3} \\
s &= \sqrt[3]{1+c+\sqrt{c^2+2\cdot c}} \\
P &= \frac{F}{3\cdot\left(s+\frac{1/s}{+}1\right)^2\cdot G^2} \\
Q &= \sqrt{1+2\cdot e^4\cdot P} \\
r_0 &= -\frac{P\cdot e^2\cdot r}{1+Q}+\sqrt{\tfrac{1}{2}\cdot a^2\left(1+\frac{1}{Q}\right)-\tfrac{P\cdot(1-e^2)\cdot z^2}{Q\cdot(1+Q)}-\frac{1}{2}\cdot P\cdot r^2} \\
U &= \sqrt{z^2+(r-r_0\cdot e^2)^2} \\
V &= \sqrt{z^2\cdot(1-e^2)+(r-r_0\cdot e^2)^2} \\
z_0 &= \frac{b^2\cdot z}{a\cdot V} \\
h &= U\left(1-\frac{b^2}{a\cdot V}\right) \\
\phi &= \arctan\left(\frac{z+e^2\cdot z_0}{r}\right) \\
\lambda &= \arctan\left(\frac{y}{x}\right)
\end{aligned}
\tag{5.17}
$$

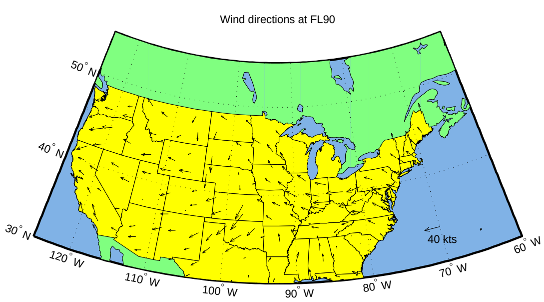
However, if working with Simulink, it will not be necessary to program these equations since there is a block already designed to do it (See Annex B).

5.5. Wind Modeling

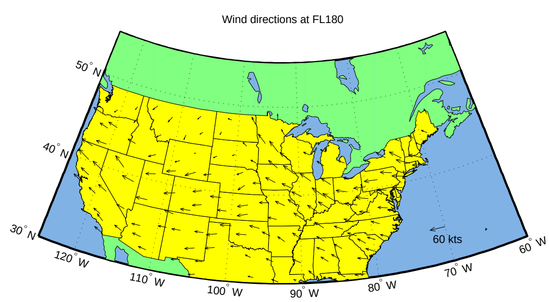
To study the aircraft trajectory, wind needs to be considered. In order to reduce the fuel consumption, the aircraft flies in a constant or optimal Mach speed. This means that, at constant altitude, the aircraft flies at constant airspeed. Supposing that there is no wind, the aircraft speed relative to the wind is the same than the aircraft speed relative to the ground. However, if there is wind, and usually it is not a negligible factor at cruise altitudes, the relative speed to the ground function of aircraft airspeed, $\vec{V_{TAS}}$, and wind speed, $\vec{W}$, is expressed in Equation 5.18.

$$\vec{GS} = \vec{V_{TAS}} + \vec{W} \tag{5.18}$$

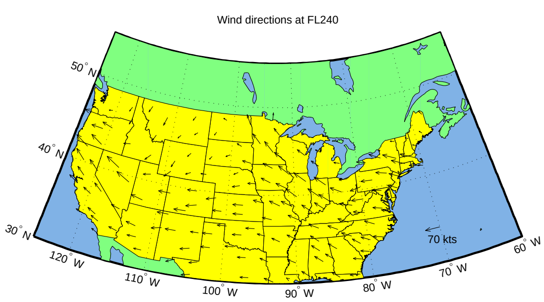
It is obvious that we can not predict the wind speed and wind direction in any place at any moment. However, the US department of Commerce through the National Oceanic and Atmospheric Administration (NOAA) publishes the wind data [Commerce, 2015] for 175 airports across This agency provides the wind speed, wind direction and air temperature. This data is provided for each airport at several altitudes from 3000 ft to FL390. With this data, wind is obtained in several parts across North America. However, this data does not form an uniform grid (Figure C.2). Therefore, interpolating with the position will not be easy.



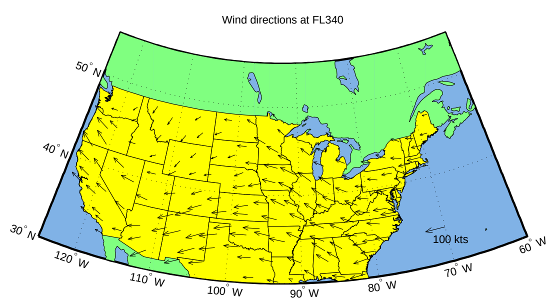
(a) Wind representation at FL90



(b) Wind representation at FL180



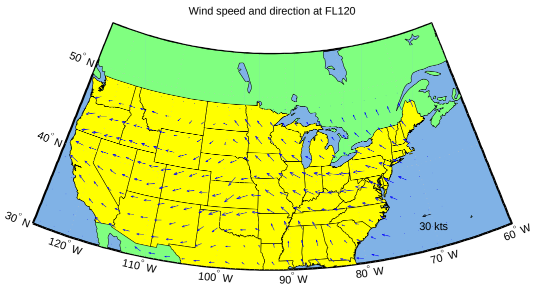
(c) Wind representation at FL240



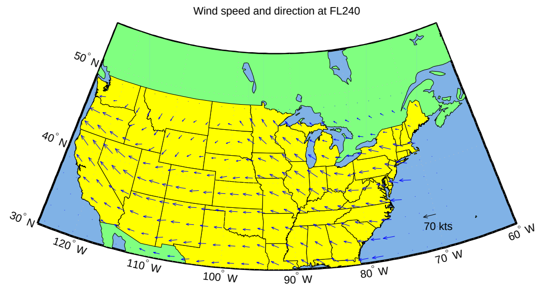
(d) Wind representation at FL340

Figure 5.1: Wind representation with non uniform data

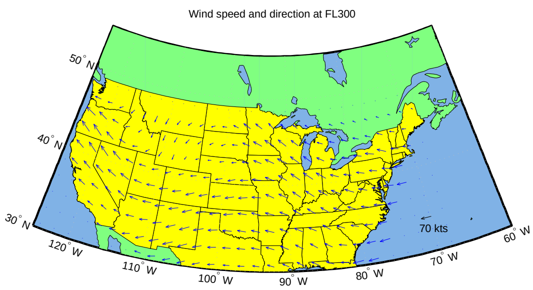
With any computing program it is possible to fill the empty spaces and fit the data to a surface. This process has been explained in Appendix C. Figure C.3 show the wind used for the simulations done in this project. This wind data dates from March 24th, 2015.



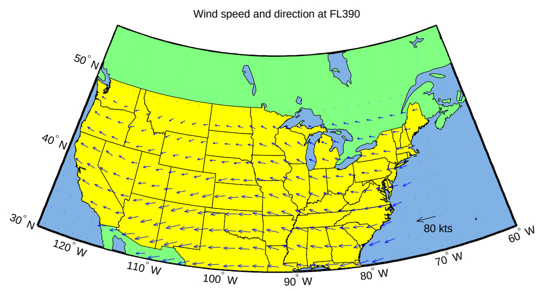
(a) Wind representation at FL60



(b) Wind representation at FL120



(c) Wind representation at FL240



(d) Wind representation at FL340

Figure 5.2: Wind representation with uniform data

5.6. Trajectory

During the simulation, the aircraft needs to be guided and, since this project focus on the cruise part of the trajectory, two real flight plans will be simulated. Data has been extracted from Flightaware website [Flightaware, 2015] from flight AC791, an Air Canada daily flight from Toronto to Los Angeles that uses the Boeing 767-300ER. Two different routes, which date from March 31st and 24th have been simulated (Figure 5.3). These consist on several waypoints where the aircraft heading changed substantially.

As Equations 5.1 to 5.6 state, ψ will be the responsible of the aircraft direction. To compute the aircraft heading between its actual position and its next waypoint Equation 5.19 has been used. λ and ϕ are the latitude and longitude coordinates of the actual position (1) and next waypoint (2).

$$\psi = \arctan \frac{\cos\phi_1 \cdot \sin\phi_2 - \sin\phi_1 \cdot \cos\phi_2 \cdot \cos\Delta\lambda}{\sin\Delta\lambda \cdot \cos\phi_2} \tag{5.19}$$

The distance to go, d, can be computed using Haversine's equation in function of the Earth's radius, which has been assumed as 6371 km (Equation 5.20).

$$\begin{aligned} a &= \sin^2\frac{\Delta\phi}{2} + \cos\phi_1 \cdot \cos\phi_2 \cdot \sin^2\frac{\Delta\lambda}{2} \\ c &= 2 \cdot \arctan\frac{\sqrt{1-a}}{\sqrt{a}} \\ d &= R \cdot c \end{aligned} \tag{5.20}$$

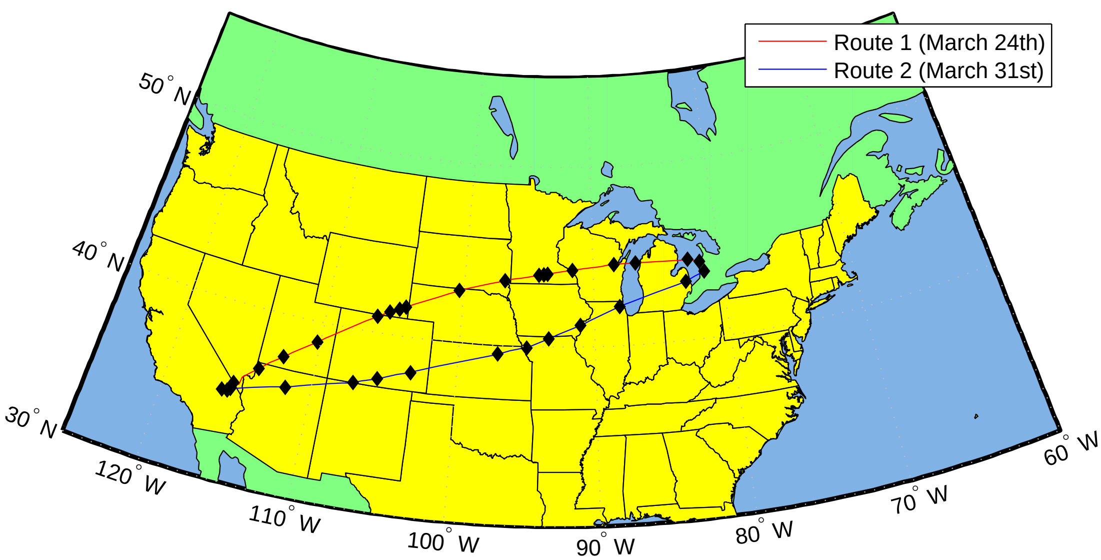


Figure 5.3: Aircraft trajectory simulations

CHAPTER 6. SIMULATION ANALYSIS RESULTS

In this section, the analysis of the simulation results obtained will be carried out. First of all, a comparison is carried out between the two different flight route patterns that AC791 usually performs (Figure 5.3). Then, the two routes will be compared considering and not considering wind. Finally, the three aerodynamic models explained above will be compared focusing mainly in the fuel consumption. To carry out all these comparisons, 12 different simulations have been carried out: for each aerodynamic model (3), for each route (2) and considering and not considering wind (2). Next list show all simulations carried out.

- DATCOM Model
  - March 24th
    - Considering wind
    - Not considering wind
  - March 31st
    - Considering wind
    - Not considering wind
- BADA Model
  - March 24th
    - Considering wind
    - Not considering wind
  - March 31st
    - Considering wind
    - Not considering wind
- Dietrich Model
  - March 24th
    - Considering wind
    - Not considering wind
  - March 31st
    - Considering wind
    - Not considering wind

Data for the flight plans has been extracted from Flightaware [Flightaware, 2015]. It provides some data regarding to nearly all the commercial flights in real-time. This website gets this data by having ADS-B spread around the world. Moreover, data regarding to the same flight during the last 5 months is also available. This data set consists of:

- Date and time.
- Aircraft model and registration number.
- Latitude and longitude coordinates for the entire flight path.
- Airspeed, vertical speed, altitude and heading.

6.1. Route analysis

In this section, both flight routes have been compared. Regarding the flight routes followed during the month of March 2015, two route patterns can be identified. One of them goes north of the great circle distance (Route 1) and the other goes south (Route 2).The minimum distance route is plotted (Figure 6.1) between both airports while the routes simulated are only plotted during the cruise since it is the analyzed part. This is the reason because both routes depart from a different point but they are not really far from each other since in those points, the standard procedures for climbing end and the standard procedures for descending begin.

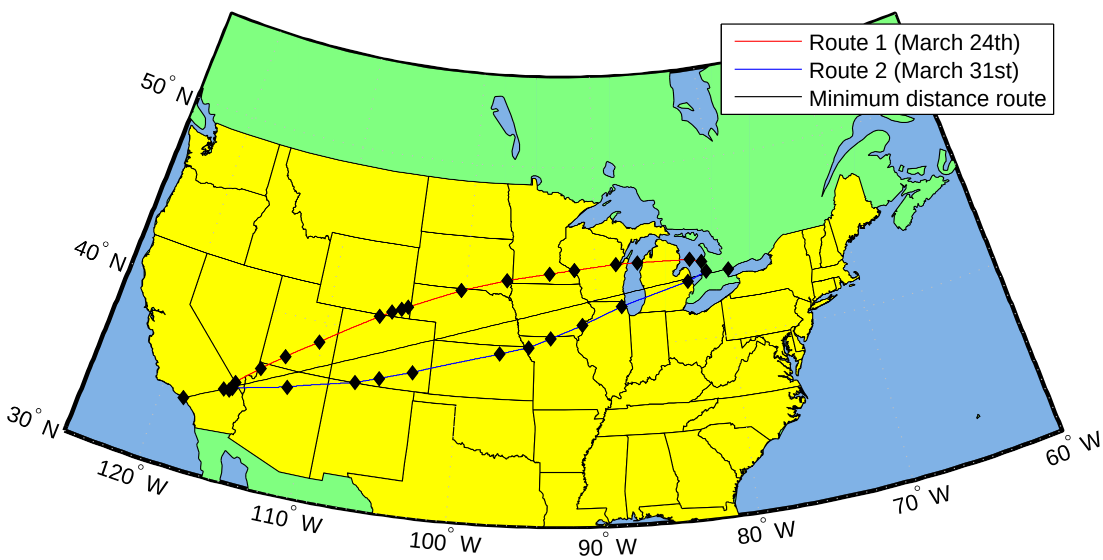


Figure 6.1: Aircraft trajectories simulated compared minimum distance route

Table 6.1 shows the results after simulating both routes considering wind.

Route	Route 1 (March 24th)	Route 2(March 31st)
Total fuel consumption $[kg]$	14526	13613
Mean fuel rate consumption $[kg/h]$	4516	4530
Total cruise time $[h]$	3:12	3:00
Total distance covered $[km]$	2919	2913
Mean airspeed $[knots]$	463	463
Mean ground speed $[knots]$	490	523

Table 6.1: Route comparison results

First of all, the total fuel consumption is a 6.7% higher for Route 1. This can be explained by the difference between both ground speeds (which is also 6.7%), which means that the aircraft has suffered more favorable winds in Route 2 than in Route 1 since the airspeed for both routes is the same. It is coherent with Figures C.2 and C.3 that show higher westerly winds for central US than north US. The difference can not be explained by the difference in total distance covered since it is negligible. Here it is possible to realize the importance of wind, which will be analyzed deeper in the following section.

6.2. Wind importance analysis

In this section, both routes have been simulated considering and not considering wind in order to understand the importance of this consideration. Table 6.2 shows the results after simulating both cases.

Route	Route 1		Route 2	
Wind	Yes	No	Yes	No
Total fuel consumption $[kg]$	14526	15950	13613	15325
Mean fuel rate consumption $[kg/h]$	4516	4495	4530	4504
Total cruise time $[h]$	3:12	3:33	3:00	3:24
Total distance covered $[km]$	2919	2919	2913	2913
Mean airspeed $[knots]$	463	463	463	463
Mean ground speed $[knots]$	490	463	523	463

Table 6.2: Wind importance analysis

Table 6.3 shows the percentage difference with Route 1 considering wind. It is possible to see that considering wind is really important. If wind is not considered, results will be far from real. Most of times winds are higher in the east earth axis than in the north earth axis, therefore for this flight it is really important since the difference on longitude for this route is higher than the difference on latitude. The fact that in this case the fuel consumption is reduced , it is caused because the aircraft is flying downwind. However, in the opposite direction, the results would be the opposite. Therefore, considering wind does not mean than the fuel consumption (and therefore, the other parameters) will be lower but than the fuel consumption will be different.

Route	Route 1	Route 2
Total fuel consumption (%)	9.81	11.79
Mean fuel rate consumption (%)	-0.47	-0.56
Mean ground speed (%)	-9.36	-12.46
Total cruise time (%)	10.32	12.34

Table 6.3: Wind importance analysis comparison

6.3. Aerodynamic coefficient model analysis

In this sections, the three aerodynamic models described in Chapters 2, 3 and 4 are compared.

First of all, as the main goal for trajectory optimization is the fuel consumption reduction, the fuel consumption results for the different models are shown in Figure 6.2. This figure clearly shows a significant difference between the Dietrich model and the other two models. Is it also possible to see that fuel consumption decreases along the flight. This only proves that the model is coherent in this aspect because, since the fuel burnt along the flight results in an aircraft weight reduction, the aircraft needs less thrust to keep going at the same airspeed and therefore the fuel consumption rate is lower.

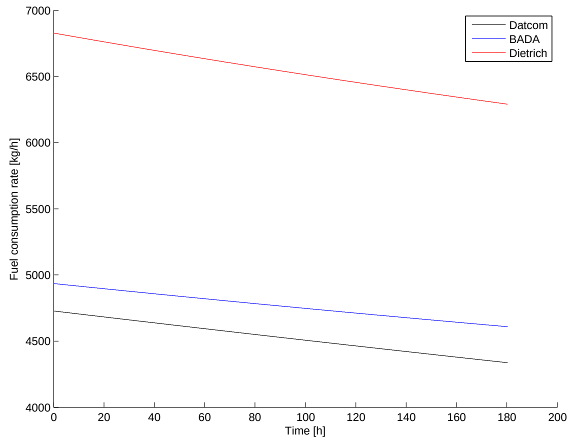
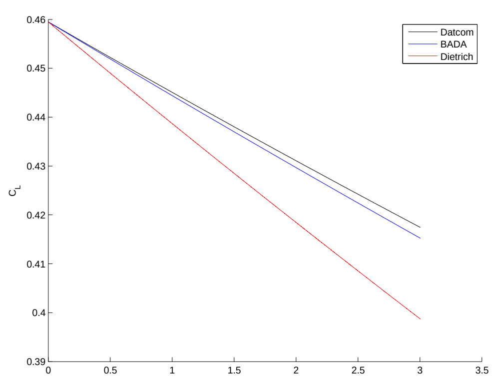


Figure 6.2: Fuel consumption rate comparison

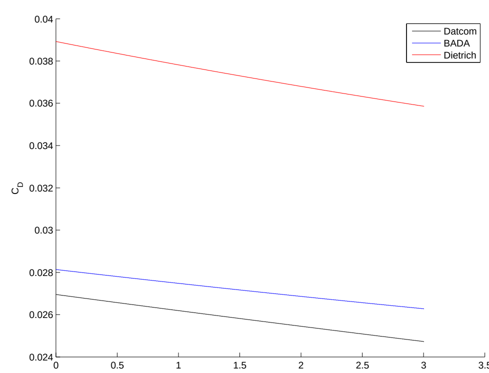
To understand the significant difference in results between the Dietrich model and the other two models, it is necessary to look at the lift coefficient and drag coefficient values obtained for these simulations (Figure 6.3). There, it is possible to see that C_L decreases as the flight progresses (Figure 6.3(a)). Therefore, the drag coefficient, which depends on the C_L, also decreases. This can be understood by knowing that C_L is obtained from the lift equation (Equation 6.1), that during cruise flight the flight path angle, γ, is zero and that $L = mg \sin\gamma$. Since the mass decreases because of the fuel burnt along the flight, the lift required is smaller and therefore drag coefficient is also smaller. This may be explained by the strong relation between drag and lift coefficients shown in Chapter 4.

$$L = \frac{1}{2}\rho V^2 S C_L \tag{6.1}$$

Regarding to the C_D (Figure 6.3(b)). it is possible to realize that from the beginning the Dietrich model C_D is higher than C_D obtained with BADA and DATCOM models. For this reason the fuel consumption will be higher, therefore the aircraft mass will be reduced faster and for this reason the C_L required for the aircraft will be less. With this, it is possible to understand why the Dietrich Model C_L decreases significantly faster than the others and why the fuel consumption rate for the Dietrich model is significantly higher.



(a) Lift coefficient C_L



(b) Drag coefficient C_D

Figure 6.3: Aerodynamic coefficients comparison

After doing some research looking for common Boeing 767-300ER cruise rate consumption values to compare them with the obtained ones, it has been found that these values vary from 3850 to 5200 kg/h [Jenkinson et al., 1999]. Also, some internet airline pilot forums like [Airliners, 2001] and [Network, 2010] say that Boeing 767-300ER cruise rate consumption is between 4.300 kg and 5.200 kg. It has also been found, from an official Boeing report [Conklin and de Decker Aviation, 2007], that the Boeing 767-200ER, which is the previous version of Boeing 767-300ER, and it is really similar has an average fuel consumption around 5.200 kg considering all phases of flight. Therefore, it is possible to conclude that BADA or DATCOM models are more accurate in their fuel consumption estimations than Dietrich Model. This might be explained because Dietrich model has been mainly developed for the Boeing 747-100 and then, it has been adapted for the Boeing 767-300ER in this project. The BADA model might still be the most accurate model since it is a complete model, including a fuel consumption estimation, and not only an estimation of one or two aerodynamic forces as DATCOM or Dietrich models.

CHAPTER 7. CONCLUSIONS

In this chapter, the conclusions of this project are presented. First of all, a brief summary of what has been done in each chapter and the relevant conclusions from each chapter are presented. Then, the main conclusions combining all the chapters, results and partial conclusions are announced. Finally, the different aspects where all this research could be continued and improved are mentioned.

7.1. Summary and partial conclusions

In Chapter 2, DATCOM software is presented. Then, the input and output parameters that DATCOM uses are shown an explained. Finally, a comparison between the two aircraft designed with DATCOM, the Boeing 767-300ER and the Boeing 777-300ER, have been compared with data available from an Airbus 320. All the parameters from both aircraft have been easily extracted from their official 3D drawings except the airfoil. This parameter has been almost arbitrarily fixed by trying to find consistent values of the aerodynamic coefficients.

In the same chapter, it has also been shown that lift coefficient hardly varies with altitude while the drag coefficient varies slightly. Regarding to the Mach number, it has been possible to show that the lift and drag coefficients decrease as Mach number increases. Usually, both causes of variation, Mach number and altitude, are not considered in traditional equations since these coefficients are considered to be only function of the aircraft geometry. Actually these variations are insignificant if compared with the aircraft geometry importance. However, it is important not to forget that speed and altitude are really important parameters when talking about aerodynamic forces, but their importance is expressed in its traditional equations (Equation 3.2 and 3.4) through the density and airspeed.

In Chapters 3 and 4, the BADA and Dietrich aerodynamic models are presented in order to understand the comparison with the DATCOM aerodynamic model that will be carried out in the following chapter.

Then, in Chapter 5, the trajectory modeling process has been presented. First of all, the set of equations that define the aircraft performance has been presented. Since these equations are affected by the atmospheric conditions, the ISA model has been briefly described. This set of equations considers mass variation, and therefore the BADA consumption model has been presented. The WGS84 datum is used to convert the cartesian coordinates, used in the 3D point aircraft model, to geodetic coordinates. The geodetic coordinates are really important for two aspects. First, they are important to be able to interpret the wind data available and second, they are important in order to guide the aircraft through setting the aircraft heading to perform the desired route. These two aspects have also been presented in the last part of this chapter.

Finally, in Chapter 6 the results of the different simulations have been analyzed by doing three different comparisons. First of all, both AC791 route patterns have been analyzed. Then, the importance of wind in aircraft trajectory simulations has been studied. Finally, the three aerodynamic coefficient models explained in the previous chapters have been analyzed and compared.

In this same chapter, some partial conclusions can be extracted. First of all, it has been possible to see that the chosen route is really important for airlines regarding to fuel consumption, and consequently for their expenses. Then, it has been shown that wind is a really important factor when dealing with aircraft trajectory assessment. Finally, after comparing the three models it has been possible to see that Dietrich Model consumption values are far from the other two models. This is caused because of significantly higher values of the drag coefficient.

7.2. Global conclusions

After summarizing all the research carried, all concepts can be unified to reach some overall conclusions.

First of all, it has been concluded that wind plays a really important role in aircraft trajectory assessment. For this reason, misleading its importance by not considering it is absolutely discouraged.

Finally, it can also be concluded that to develop an aircraft performance model it is recommended to develop the whole aircraft model and not only some parts of the aircraft performance because then, it will be difficult to improve the current existing performance models. However, it has been proven that DATCOM consumption estimations are not really far from the fuel consumption values found in the literature.

7.3. Future research

Despite of the fact that a lot of research has been done regarding aircraft trajectory modeling, there is still a lot to do and investigate. Some of the fields where the research can be continued are explained in this section.

First of all, the main issue of this project has been not having the real aircraft airfoil data or its estimation and not being able to estimate it. Being able to provide airfoil data to DATCOM would lead to be absolutely sure that the results provided by DATCOM are totally reliable.

Also, DATCOM provides a wide number of outputs to determine all the aerodynamic forces and moments that there are during an aircraft actuation. Considering all the factors, including the dynamic derivatives, could also provide really interesting results.

Another point where the research should be focused it, would be the whole trajectory assessment and not only the cruise part. Although the cruise part is the most flexible part when talking about optimization, the other flight phases as take-off and approximation should be considered. Although these phases are really limited by operational constraints, there is still some optimization margin.

Finally, although in this project the aircraft is considered to be a point mass model, it should be considered a 3D body where forces and moments act on the three axis and aircraft deflection surfaces play a really important role.

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APPENDICES

APPENDIX A. AIRCRAFT MODELING WITH DATCOM

In this appendix the DATCOM commands to model Boeing 767-300ER and Boeing 777-300ER is provided.

```
 Author: Joaquim Villen Benseny
 Last Modification: April 4th 2015
 Description: DATCOM parameters for Boeing 767-300ER.
 Reference Document: Bachelor Thesis
************************
*   List of Command Card
************************
DIM FT
DAMP
DERIV RAD
PART


**********************
*  Flight Conditions *
**********************

 $FLTCON NMACH=9.0, MACH(1)=.1,.2,.3,.4,.5,.6,.7,.8,.86,
         NALT=20.,ALT(1)=0.0,35.0,100.0,1000.0,1500.0,3000.0,10000.0,12000.0,
         14000.0,18000.0,20000.0,24000.0,28000.0,30000.0,
         32000.0,34000.0,36000.0,38000.0,40000.0,44000.0,
         NALPHA=20.0,
         ALSCHD(1)= -16.0, -8.0, -6.0, -4.0, -2.0, 0.0, 2.0, 4.0, 8.0, 9.0,
             10.0, 12.0, 14.0, 16.0, 18.0, 19.0, 20.0, 21.0, 22.0, 24.0,
         GAMMA=0., LOOP=2.0,STMACH=0.99,$




**************************************
* Group II     Synthesis Parameters  *    page 33
**************************************
 $SYNTHS XCG=92.2,ZCG=0.0,
        XW=56.3,ZW=-1.1,
        ALIW=1.0,
        XV=143.6,ZV=4.67,
        XH=149.7,ZH=1.4,
        VERTUP=.TRUE.,$
**********************************
*  Body Configuration Parameters *    page 36
**********************************
 $BODY NX=20.,
    X(1)=0.,2.74,4.18,5.92,8.31,12.17,
       15.98,19.61,25.19,120.91,128.74,
       135.83,141.71,148.79,153.62,159.04,166.88,171.55,
       174.57,175.06,
    ZU(1)=.04,1.98,3.36,4.65,5.86,7.10,7.85,8.24,
       8.34,8.34,8.34,8.33,8.23,7.71,6.99,6.18,
```

```
       4.93,3.98,2.65,2.37,
    ZL(1)=-3.61,-5.15,-5.78,-6.46,-7.24,-8.22,-8.83,-9.20,
       -9.20,-9.40,-9.11,-8.31,-7.19,-5.66,-4.60,-3.42,
       -1.70,-0.61,.85,1.22,
    S(1)=9.40,38.35,59.68,85.73,119.98,165.79,210.54,
         220.28,226.54,226.54,226.05,212.19,184.78,
         147.40,111.71,69.43,33.51,15.81,2.00,0.97,$


 $WGPLNF CHRDR=37.8,CHRDTP=6.9,CHRDBP=24.3,
    SSPN=77.6,SSPNOP=53.2,SSPNE=69.4,CHSTAT=.25,
    TWISTA=-3.,TYPE=1.,SAVSI=33.6,SAVSO=33.6,DHDADO=6.$
 $VTPLNF CHRDR=23.7,CHRDTP=7.7,SAVSI=44.,
    SSPN=36.3,SSPNOP=0.,SSPNE=31.4,CHSTAT=.25,TWISTA=0.,TYPE=1.$
 $HTPLNF CHRDR=21.9,CHRDTP=5.4,
    SSPN=29.9,SSPNE=27.9,CHSTAT=.25,TWISTA=0.,TYPE=1.,
    SAVSI=38.,DHDADI=7.7$
NACA-W-5-68012-25
NACA-V-4-0012-25
NACA-H-4-2412-25


 Author: Joaquim Villen Benseny
 Last Modification: April 4th 2015
 Description: DATCOM parameters for Boeing 777-300ER.
 Reference Document: Bachelor Thesis

************************
*   List of Command Card
************************
DIM FT
DAMP
DERIV RAD
PART


*********************
*  Flight Conditions *
*********************

 $FLTCON NMACH=9.0, MACH(1)=.1,.2,.3,.4,.5,.6,.7,.8,.86,
         NALT=20.,ALT(1)=0.0,35.0,100.0,1000.0,1500.0,3000.0,10000.0,12000.0,
         14000.0,18000.0,20000.0,24000.0,28000.0,30000.0,
         32000.0,34000.0,36000.0,38000.0,40000.0,44000.0,
         NALPHA=20.0,
         ALSCHD(1)= -16.0, -8.0, -6.0, -4.0, -2.0, 0.0, 2.0, 4.0, 8.0, 9.0,
             10.0, 12.0, 14.0, 16.0, 18.0, 19.0, 20.0, 21.0, 22.0, 24.0,
         GAMMA=0., LOOP=2.0,STMACH=0.99,$




*************************************
* Group II     Synthesis Parameters  *    page 330
*************************************
 $SYNTHS XCG=131.77,ZCG=0.0,
        XW=88.2,ZW=-3.38,
        ALIW=1.0,
```

```
        XV=200.3,ZV=6.2,
        XH=206.6,ZH=2.35,
        VERTUP=.TRUE.,$
*********************************
*  Body Configuration Parameters *    page 36
*********************************
 $BODY NX=20.,BNOSE=2.,BTAIL=2.,
        X(1)=.0,1.13,3.22,5.72,9.05,13.22,21.55,25.72,31.97,174.87,179.88,
        188.22,196.55,209.05,217.38,221.55,223.63,225.72,229.88,238.22,
        ZU(1)=-1.87,-0.67,0.65,3.03,4.74,6.10,8.03,8.65,9.14,9.14,9.14,
        9.11,8.98,7.94,7.18,6.84,6.62,6.4,5.97,5.05,
        ZL(1)=-4.14,-5.20,-6.2,-7.25,-8.42,-9.50,-10.80,-11.08,-11.16,
        -11.16,-11.16,-10.18,-8.66,-6.1,-3.21,-2.7,-2.25,-1.28,0.92,
        S(1)=3.47,14.36,35.73,73.71,121.17,173.98,268.78,302.09,
        323.85,323.85,323.85,296.72,246.52,149.02,86.46,65.88,60.09,
        50.44,30.19,1.40,$
*       P(1)=13.5,21.2,30.5,39.2,46.9,58.1,61.6,63.8,63.8,63.8,61.1,55.7,
*       43.3,33.2,29.0,27.6,25.3,19.9,9.2,$


 $WGPLNF CHRDR=49.9,CHRDTP=7.1,CHRDBP=27.0,
    SSPN=101.0,SSPNOP=65.9,SSPNE=91.3,CHSTAT=.25,TWISTA=-3.,TYPE=1.,
    SAVSI=33.6,SAVSO=33.6,DHDADO=5.3$
*****************************************
*         Vertical Tail planform variables   pg 37-38
*****************************************
 $VTPLNF CHRDR=27.,CHRDTP=7.9,SAVSI=44.5,
    SSPN=40.1,SSPNOP=0.,SSPNE=33.5,CHSTAT=.25,TWISTA=0.,TYPE=1.$

********************************************
*         Horizontal Tail planform variables   pg 37-38
********************************************
 $HTPLNF CHRDR=21.8,CHRDTP=7.7,
    SSPN=34.8,SSPNE=32.4,CHSTAT=.25,TWISTA=0.,TYPE=1.,
    SAVSI=38.,DHDADI=8.$

NACA-W-5-68012-25
NACA-V-4-0012-25
NACA-H-4-2412-25
```

APPENDIX B. MATLAB & SIMULINK CODE DESCRIPTION

The software code used to model the aircraft's trajectory will be presented in this appendix. Matlab® and Simulink® software has been used to develop this project. Since this project could still be developed deeper, the main goal of this appendix is to explain all the software code used to make it easy if anyone ever is carrying out deeper research in this case.

B.1. Matlab Code

The Matlab Software is the main code used.

```
% Author: Joaquim Villen Benseny
% Last Modification: April 4th 2015
% Description: Computes the consumption of a B763ER flight.
% Reference Document: Bachelor Thesis
% Important outputs (all function of time):
%        -   x: east coordinate position (east>0). Reference: initial
%        position. Units: meters
%        -   y: north coordinate position (north>0). Reference: initial
%        position. Units: meters
%        -   alt: altitude WGS84. Units: meters.
%        -   latlon: 2D vector with latitude and longitude WGS84
%        -   positions. Units:
%        degrees.
%        -   massVariation: aircraft's weight variation. Units: Kg.
%        -   psi: aircraft's heading. Units: degrees.
%        -   dm: mass rate variation (dmass/dt) Units: Kg/s
%        -   UwVw: 2D vector with wind speed values along east and north
%        axis affecting to the aircraft trajectory. Units: m/s
%% Constant parameters
SLGravity=9.80665;
%% Load data
load('dataDatcom_8Feb_V2.mat'); %Datcom data
load('reducedCruiseFlightPlan31March'); % Flight Plan (flightaware.com)
windStruct=load('windForInterpolating.mat'); % Wind data
%% BADA Data for CD
CD0=.18000E-01;
CD2=.48000E-01;
%% B763-data For improved CD
%Geometrical data
%http://www2.ita.br/~bmattos/mundo/country/usa/boeing-767.html
S = 283.3 ; % surface
b = 47.6 ; % wing span
lambda = b*b/S ; % taper ratio
tc_root = 0.151 ; % thickness/chord ratio at root
tc_tip = 0.103 ; % thickness/chord ratio at tip
sweep2 = 31*pi/180 ; % half-chord sweep angle - sweep2 = sweep4
sweep4 = 31 *pi/180; % half-chord sweep angle (degres)
sweep = 31.5*pi/180 ; % half-chord sweep angle - sweep = sweep4
l = 54.9 ; % body length
```

```
d = 5.2 ; % body diameter
b = 47.6 ; % wing span
kappa = 2*pi/(1+2/lambda)/(6.28 + 4.7*tc_root);
eprim = (1-0.045*lambda^(0.068))*(1-0.227*sweep4^(1.615));
% Oswald Coefficient only for wing
e = eprim*(1-(d/b)^2); % Oswald coefficient for the whole aircraft
mac=S/b; % mean aerodynamic chord
lnose= 15*0.3048; % Estimated
ltail = 40*0.3048; % Estimated
ka=0.95; %Korn factor
% http://adg.stanford.edu/aa241/drag/wettedarea.html
Swet_wing = 2*(1+0.2*tc_root)*S;
Swet_nose = 0.75*pi*d*lnose;
Swet_tail = 0.72*pi*d*ltail;
Swet_const = d*pi*(l-lnose-ltail);
Swet_body = Swet_nose + Swet_tail + Swet_const;
Sw=283.3; % Wing surface
%% Engine BADA
Cf1=0.79100; % Bada Engine coefficients
Cf2=0.28100e4;
Cf3=.179e2;
Cf4=.836e5;
Cfcr=.10347e1;

%% Initial conditions
ctMach=true; % Fly Constant Mach
h0=distdim(34000,'ft','m'); % initial altitude
x0=0; % Initial position x,y
y0=0;
mach=0.8; % Cruise mach
[T0,a0,p0,rho0]=atmosisa(h0); % atmosferical conditions ISA
v0=a0*mach; % Airspeed
gamma0=0; % Initial flight Path angle
psi0=244*pi/180; % Initial heading
mass=156489*0.95; % Initial mass
CI=[x0 y0 h0 v0 gamma0 psi0 mass]; % Initial conditions
q0=0.5*rho0*v0^2; % Initial Dynamic pressure
angleInDeg=[flight.lat(1) flight.lon(1)]; % Initial Cruise position

%% Simulation parameters
modelSelector=1; % 1: Datcom 2: Bada 3: Dietritch
windSelector=1; % 1: Wind active 2: No-Wind

sim('HorizontalVerticalDynamics_V2.slx');
```

B.1.1. Input Data

In this subsection the data loaded in Section B.1. is provided.

B.1.1.1. DATCOM Data

After executing the code

```
load('dataDatcom_8Feb_V2.mat');
```

a struct with name *aero* will be obtained. This data will be provided by an output file after running the DATCOM input file. This data from DATCOM can be imported to Matlab by running the following command:

```
datcomimport(outputFileName);
```

After running it, a struct with name *aero* will be obtained. This struct will have several fields, but next list shows the only fields that this software is going to use.

- aero.alpha: array containing the i angles of attack in degrees used for the DATCOM simulation.
- aero.mach: array containing the k Mach's numbers used for the DATCOM simulation.
- aero.alt: array containing the m altitudes in feet used for the DATCOM simulation.
- aero.cd: Set of m matrix containing the drag coefficient (C_D) depending on the angle of attack (row) and Mach number (column).
- aero.cl: Set of m matrix containing the drag coefficient (C_L) depending on the angle of attack (row) and Mach number (column).

```
aero =

                  mach: [0.1000 0.2000 0.3000 0.4000 0.5000 0.6000
                  0.7000 0.8000 0.8600]
                   alt: [1x20 double]
                 alpha: [-16 -8 -6 -4 -2 0 2 4 8 9 10 12 14 16 18 19 20 21 22 24]
                    cd: [20x9x20 double]
                    cl: [20x9x20 double]
```

B.1.1.2. Flight Plan Data

After executing the code

```
load('dataDatcom_8Feb_V2.mat');
```

a struct with name *flight* will be obtained. The *flight* struct will have 4 fields:

- fight.lat: latitude's vector in degrees (Table B.1)
- fight.lon: longitude's vector in degrees (Table B.1)
- fight.alt: altitude's vector in FL (Table B.1)
- fight.nWaypoints: integer containing the number of waypoints. In this case it would be 13.

This is an example of how the struct *flight* should look like.

Latitude (deg)	Longitude (deg)	Altitude (FL)
43.7	-81.3	340
43.3	-82.8	340
42.2	-88.1	340
41.2	-91.1	340
40.5	-93.4	340
39.9	-95.0	340
39.5	-97.1	340
38.1	-103.3	340
37.5	-105.6	340
37.0	-107.2	340
36.0	-111.8	340
35.2	-115.4	340
35.1	-115.6	340

Table B.1: Flight Plan Data

```
flight =

           lat: [13x1 double]
           lon: [13x1 double]
           alt: [13x1 double]
    nWaypoints: 13
```

B.1.1.3. Wind Data

After executing the code

```
load('windForInterpolation.mat');
```

a struct with name *windStruct* is obtained. This struct has 5 fields:

- uqMatrix: Set of k matrix containing the values of the wind speed in kts in the East axis depending on the latitude (row) and longitude (column). k is the number of altitudes for which data is available.
- vqMatrix: Set of k matrix containing the values of the wind speed in kts in the North axis depending on the latitude (row) and longitude (column). k is the number of altitudes for which data is available.
- altitudes: array of the k altitudes in feet for which data is avaliable.
- latitudesVector: array of latitudes in degrees corresponding to each matrix row.
- longitudesVector: array of longitudes in degrees corresponding to each matrix column.

This is an example of how the *windStruct* should look like.

```
windStruct =

            uqMatrix: [13x29x8 double]
            vqMatrix: [13x29x8 double]
           altitudes: [6000 9000 12000 18000 24000 30000 34000 39000]
     latitudesVector: [1x13 double]
    longitudesVector: [1x29 double]
```

How to obtain this data is explained in detail in Appendix C

B.2. Simulink Block Diagrams

In this section the Simulink Block Diagrams used to obtain the results of this project are briefly explained. Next list show how these blocks are structured:

- Main Program B.1
  1. Pre-trajectory
     (a) x, y, z to latlon
     (b) Wind Computing
     (c) Yaw Computing
  2. Lift & Drag
     (a) q
     (b) DATCOM Model
     (c) BADA Model
     (d) Improved CD (Dietrich Model)
        i. CL
        ii. CD
  3. $\dot{x}$ equation
  4. $\dot{y}$ equation
  5. Ground Speed equation
  6. $\dot{h}$ equation
  7. $\dot{V}$ equation
  8. $\dot{\gamma}$ equation
  9. $\dot{\psi}$ equation
  10. $\dot{m}$ equation

B.2.1. Main Block Diagram

Figure B.1 shows the main block diagram. This program iterates the block of differential equations included from sub-systems 3 to 10. Sub-system 1 computes the latitude and longitude coordinates, the wind speeds and the actual heading that the aircraft must follow. Sub-system 2 computes the aerodynamic forces and thrust required depending on the chosen model (DATCOM, BADA or Dietrich).

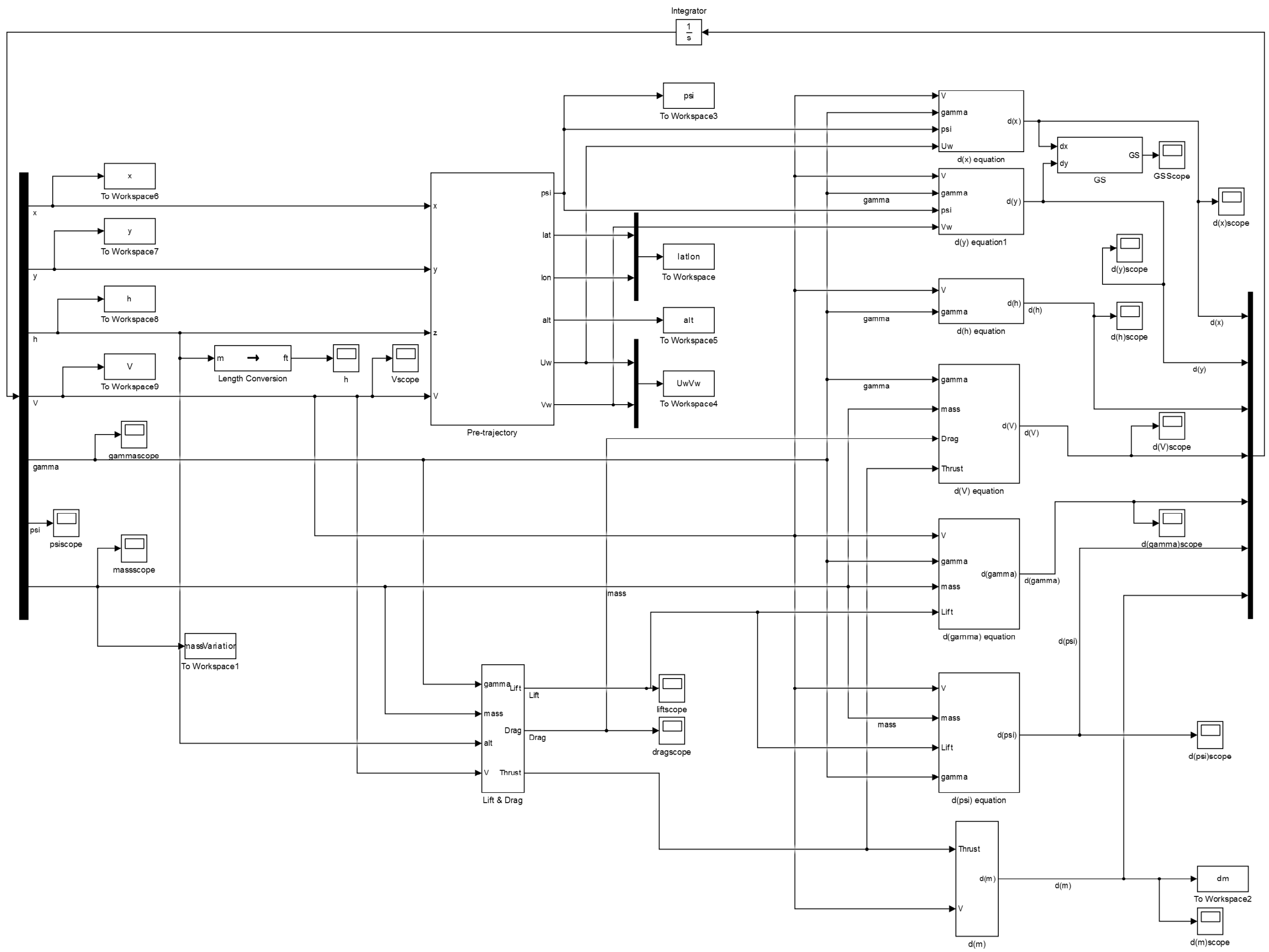


Figure B.1: Simulink Main Block Diagram

B.2.1.1. *Block's parameters*

B.2.1.2. *Sub-systems*

a Pre-trajectory

This block B.2 consists in obtaining the necessary data depending on the aircraft path. In this case, three sub-systems are used to compute the WGS84 latitude, longitude and altitude, the wind speeds and the aircraft's heading function of the flight plan.

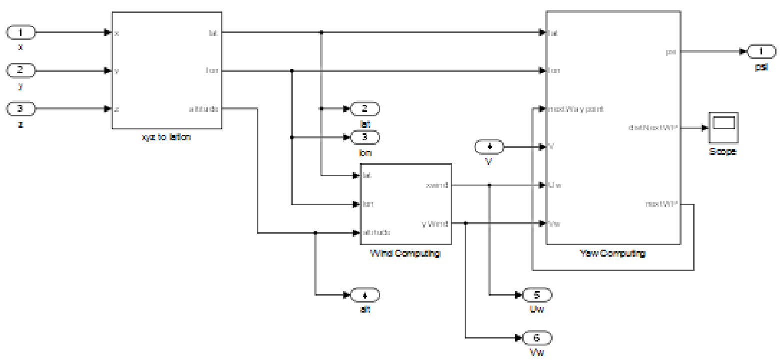


Figure B.2: Pre-Trajectory

- Inputs
    1. x: x-coordinate [m].
    2. y: y-coordinate [m].
    3. z: z-coordinate [m].
    4. V: aircraft's airspeed [m/s].
- Outputs
    1. psi: aircraft's heading [rad].
    2. lat: WGS84 latitude [deg].
    3. lon: WGS84 longitude [deg].
    4. alt: WGS84 altitude [m].
    5. Uw: East-axis wind speed [m/s].
    6. Vw: North-axis wind speed [m/s].
- Sub-systems

    1. **xyz to latlon**

    This block B.3 converts the x, y, z coordinates to WGS84 latitude, longitude and altitude. The negative gains are used to be obtain the East and North positive latitudes and longitudes respectively. The x axis is considered 90° clock-wise from north and the initial position will be the aircraft's initial position since x_0, y_0 will be considered 0.

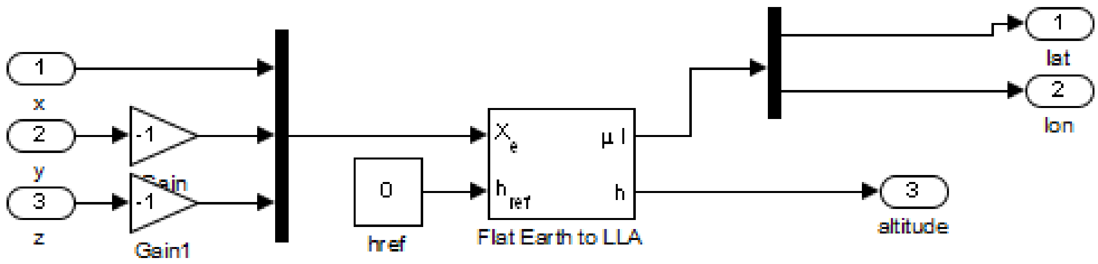


Figure B.3: xyz to latlon

- Inputs:
  (a) x: x-coordinate [m].
  (b) y: y-coordinate [m].
  (c) z: z-coordinate [m].
- Outputs:
  (a) lat: WGS84 latitude [deg].
  (b) lon: WGS84 longitude [deg].
  (c) alt: WGS84 altitude [m].

Wind Computing

This block B.4 computes the wind by interpolating a three-dimensional table depending on the latitude, longitude and altitude.

2. - Inputs
     (a) lat: WGS84 latitude [deg].
     (b) lon: WGS84 longitude [deg].
     (c) alt: WGS84 altitude [m].
   - Outputs
     (a) Uw: East-axis wind speed [m/s].
     (b) Vw: North-axis wind speed [m/s].
   - Calculations: Uses 3D interpolation. Switch makes the choice between considering or not considering the wind. If not considered, wind speeds will be 0 in both axes.

3. **Yaw Computing**

This block B.5 computes the heading of the aircraft in function of the wind speed, aircraft's airspeed, actual position and flight plan. The heading computing will consider the wind in advance to be more optimal.

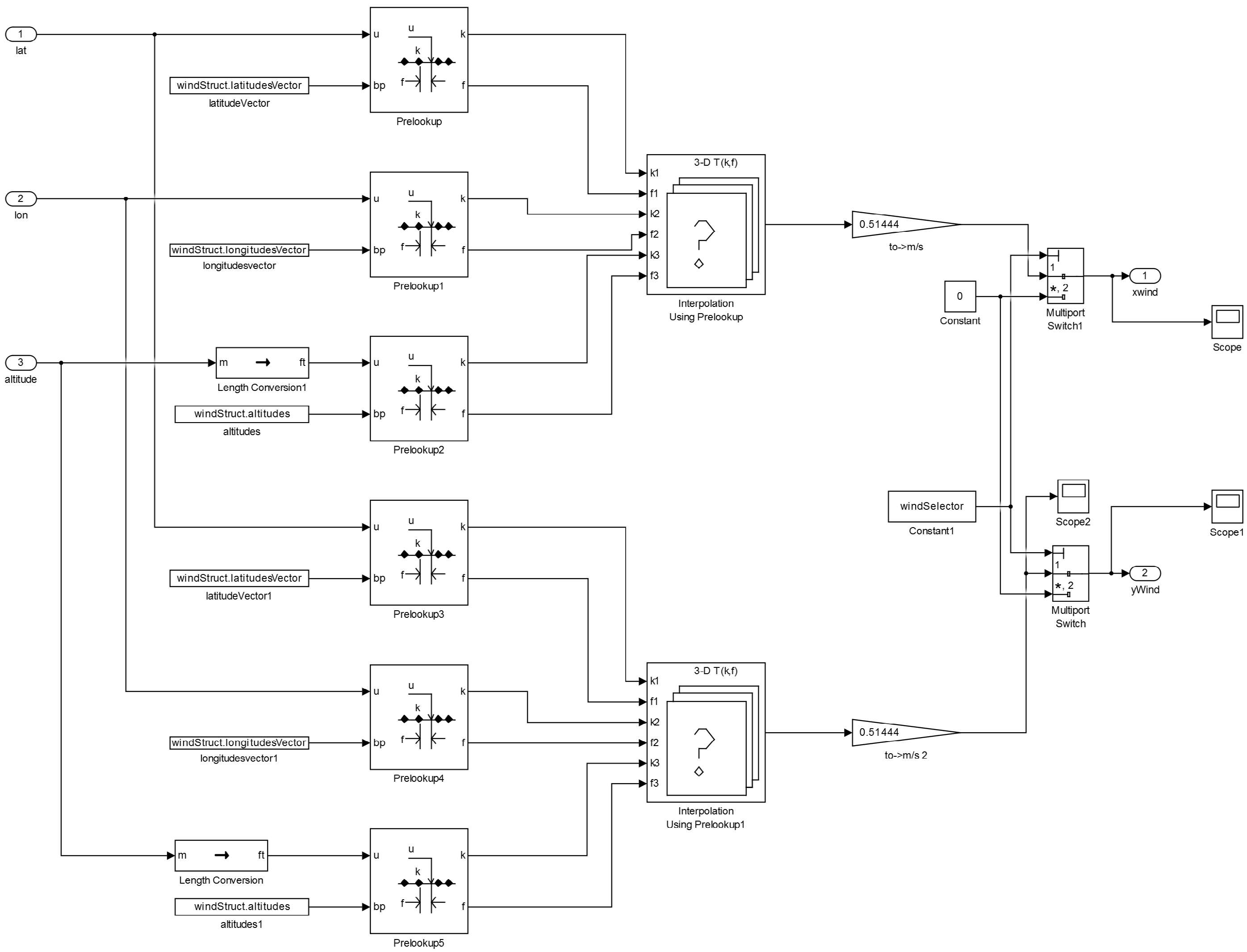


Figure B.4: Wind Computing

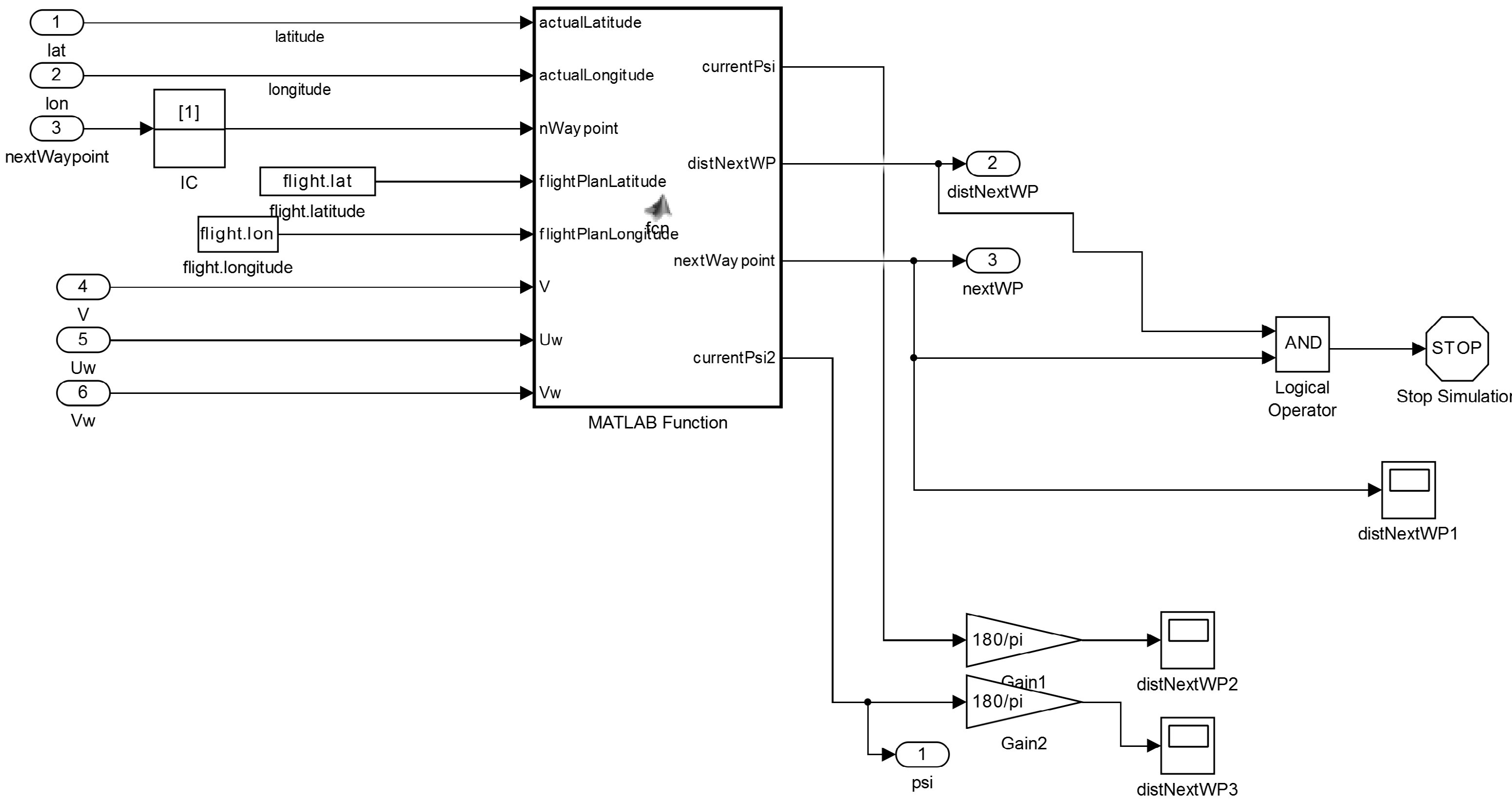


Figure B.5: Yaw Computing

- Inputs
  (a) lat: WGS84 latitude [deg].
  (b) lon: WGS84 longitude [deg].
  (c) nextWaypoint: integer containing the flight plan next waypoint the aircraft is heading to.
  (d) V: aircraft airspeed [m/s].
  (e) Uw: East-axis wind speed [m/s].
  (f) Vw: North-axis wind speed [m/s].
- Outputs
  (a) psi: aircraft heading [rad].
- Sub-systems
  (a) Matlab function (Listing B.1)

b Lift & Drag

This block B.6 computes the aerodynamic forces depending on the aircraft mass, flight path, altitude and speed using the different models presented in this project. A switch will chose which model is the program using for each simulation.

- Inputs
  1. gamma: flight path angle [rad].
  2. mass: aircraft mass [kg].
  3. alt: aircraft altitude [m]
  4. V: aircraft airspeed [m/s].
- Outputs
  1. Lift: Lift Force [N].
  2. Drag: Drag Force [N].
  3. Thrust: Thurst Force [N].

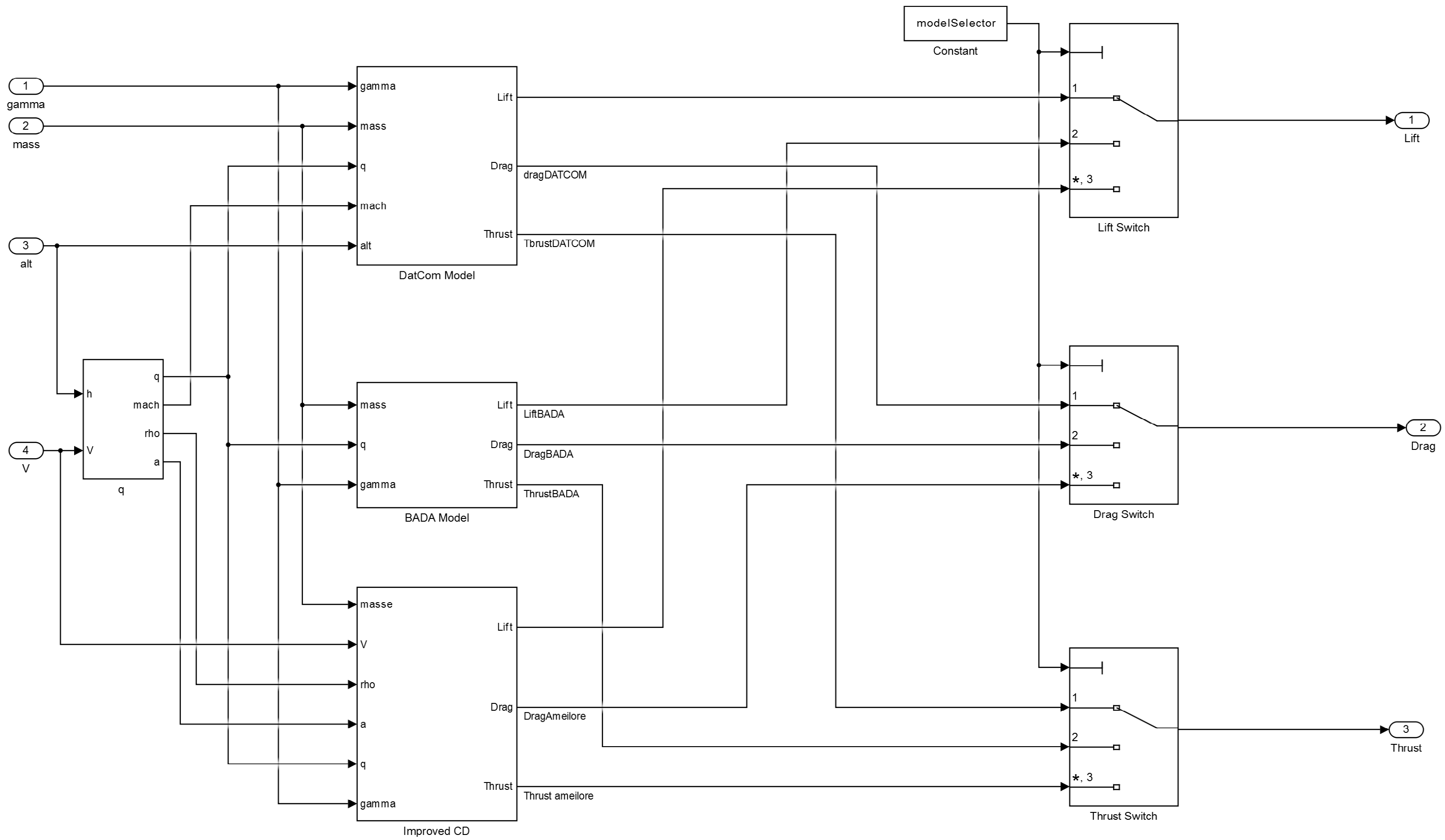


Figure B.6: Lift & Drag

- Sub-systems

1. **q**

   This block B.7 computes the atmosferical and flight conditions function of the altitude and the aircraft's airspeed.

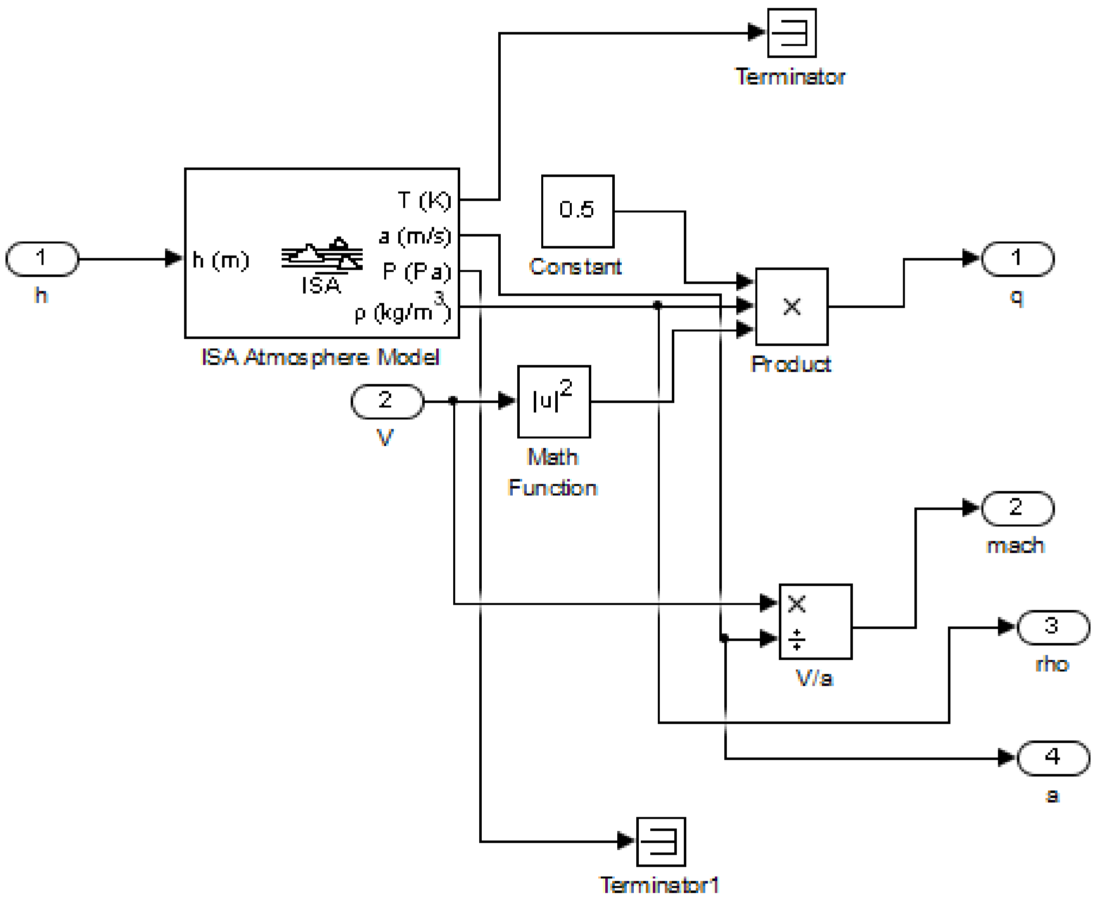


Figure B.7: q

   – Inputs
     (a) h: alt: aircraft altitude [m].
     (b) V: aircraft airspeed [m/s].
   – Outputs
     (a) q: dynamic pressure [Pa].
     (b) mach: Mach number.
   – Calculations: The ISA Athmosphere Model computes temperature, speed of sound, pressure and density at a certain altitude. Afterwards, dynamic pressure is computed with the expression $q = 0.5 \cdot rho \cdot V^2$ and Mach number using Equation 5.7.

DATCOM Model

This block B.8 computes the aerodynamics forces applied on the aircraft depending on the flight conditions using the data obtained with DATCOM.

2. – Inputs
   (a) gamma: flight path angle [rad].
   (b) mass: aircraft mass [kg].
   (c) q: dynamic pressure [Pa].
   (d) mach: Mach number.
   (e) alt: aircraft altitude [m].
   – Outputs
   (a) Lift: Lift Force [N].
   (b) Drag: Drag Force [N].
   (c) Thrust: Thurst Force [N].
   – Calculations: Lift is computed by using Equation D.10 and C_L is computed using Equation D.12. Then, the Drag Coefficient is computed by using the Matlab function B.2.This is explained more in detail in Appendix D. Then, the Thrust required is computed by using Equation D.10.
   – Sub-Systems
   (a) Matlab Function (Listing B.2).

3. **BADA Model**

This block B.9 computes the aerodynamics forces applied on the aircraft depending on the flight conditions using the BADA method.

– Inputs
  (a) mass: aircraft mass [kg].
  (b) q: dynamic pressure [Pa].
  (c) gamma: flight path angle [rad].
– Outputs
  (a) Lift: Lift Force [N].
  (b) Drag: Drag Force [N].
  (c) Thrust: Thurst Force [N].
– Calculations: Computes the C_L and C_D using Equations 3.2 to 3.4.

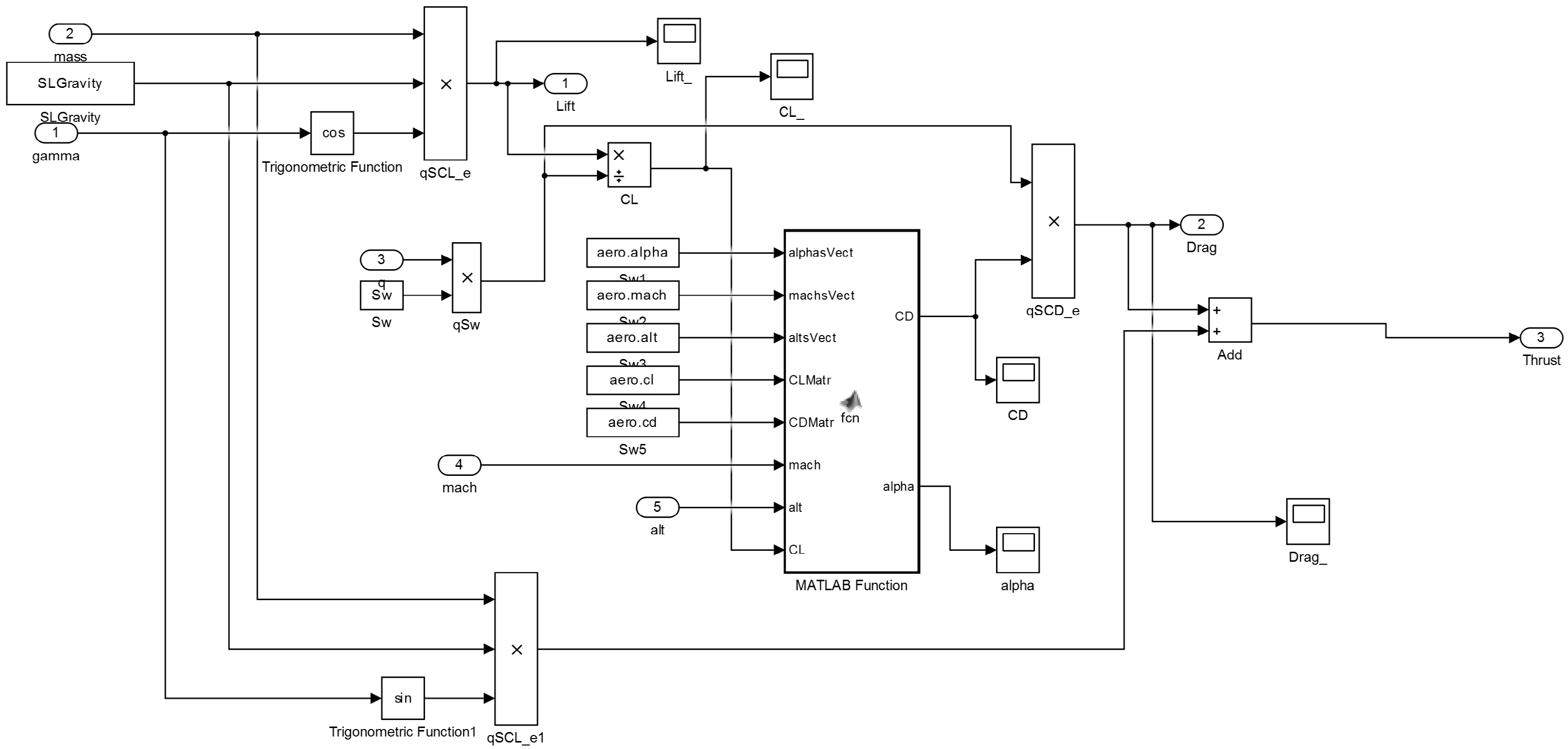


Figure B.8: DATCOM Model

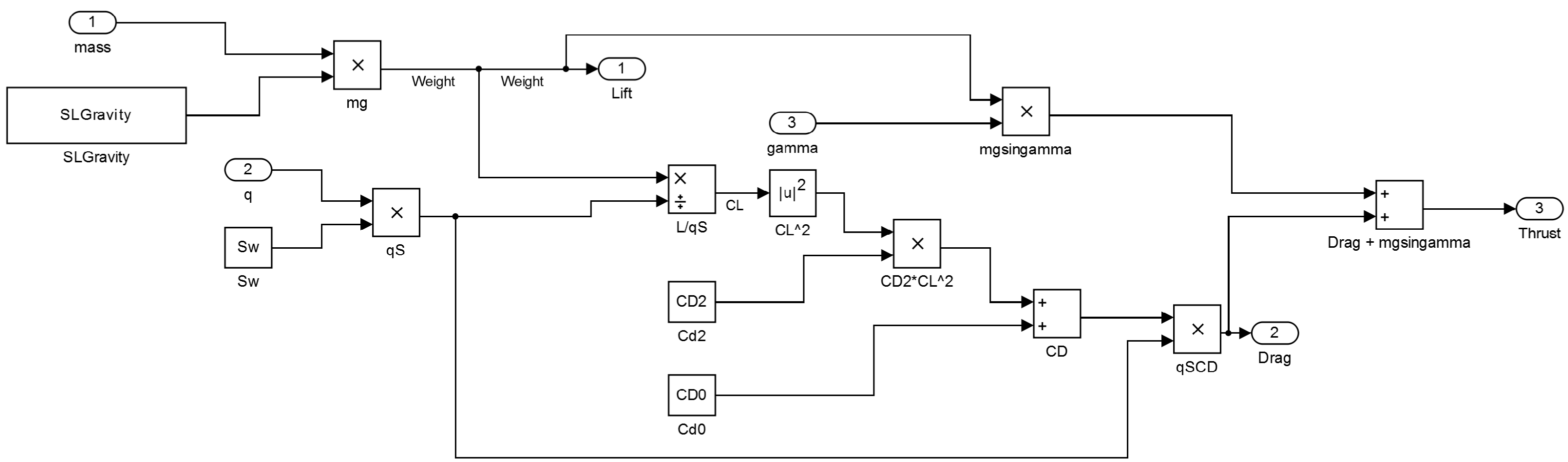


Figure B.9: BADA Model

Improved CD

This block B.11 computes the aerodynamic forces applied on the aircraft depending on the flight conditions using the Dietrich Method.

4. – Inputs
   (a) mass: aircraft mass [kg].
   (b) V: aircraft airspeed [m/s].
   (c) rho: air density [kg/m^3].
   – Outputs
   (a) Lift: Lift Force [N].
   (b) Drag: Drag Force [N].
   (c) Thrust: Thurst Force [N].
   – Calculations: Mach's number is computed using Equation 5.7. Thrust is computed using Equation D.10.
   – Sub-Systems
   (a) CL: C_L is computed using Equation 3.2.
   (b) CD: C_D is computed as explained in Section 4.2. and Equations 4.1 to 4.16.

c d(x)equation

This block B.10 computes Equation 5.5.

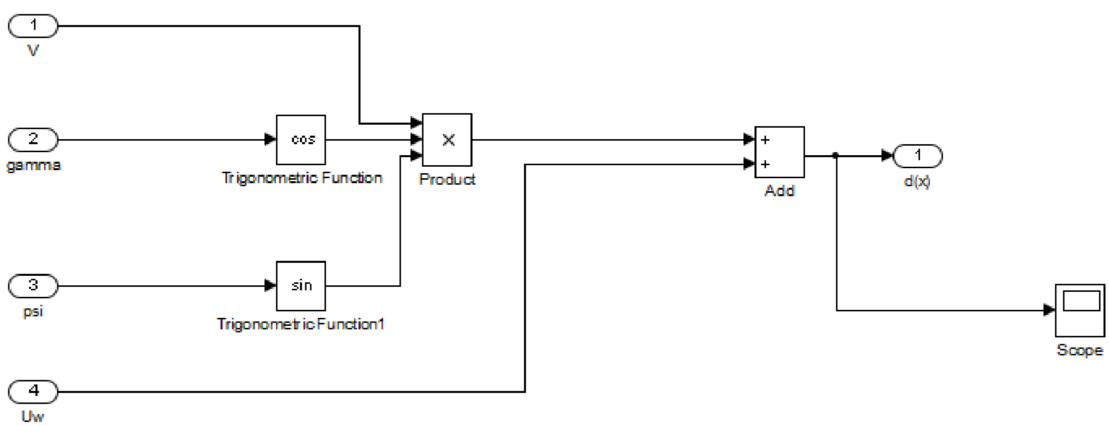


Figure B.10: d(x) equation

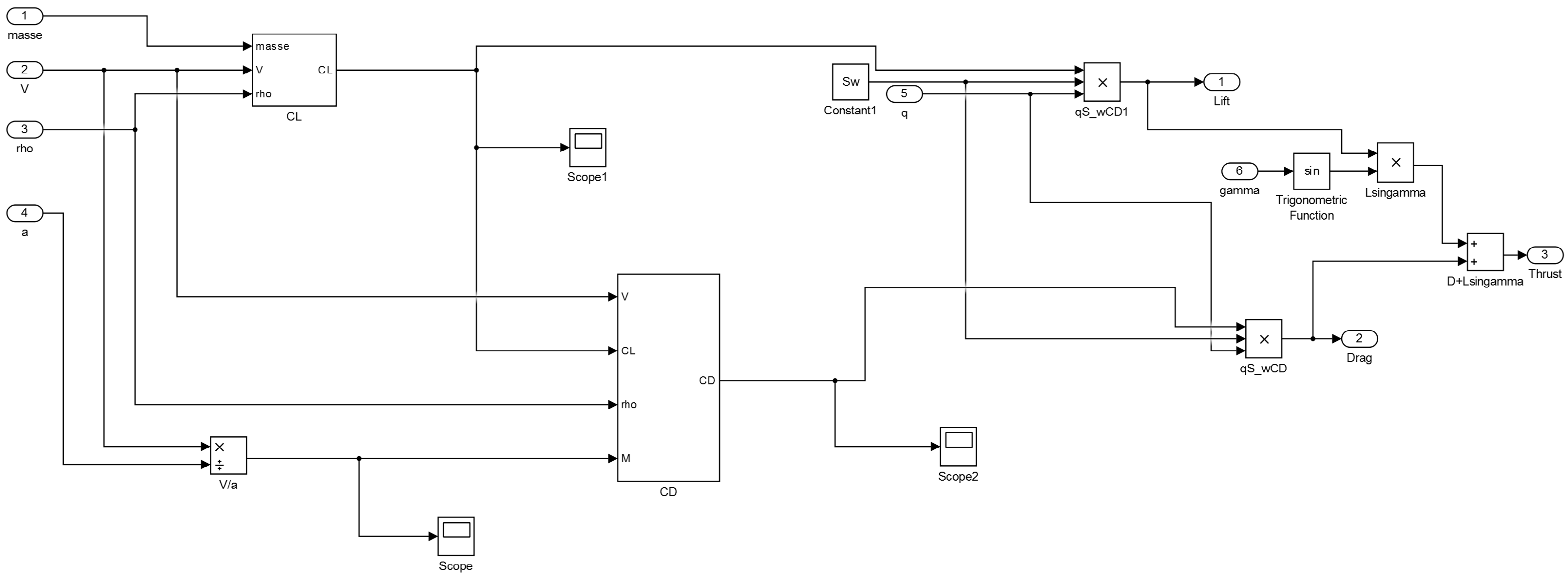


Figure B.11: Improved CD

- Inputs
    1. V: aircraft airspeed [m/s].
    2. gamma: flight path angle [rad].
    3. psi: aircraft heading [rad].
    4. Uw: East-axis wind speed [m/s].
- Outputs
    1. d(x): derivative of x-coordinate with respect to time.
- Calculations: Equation 5.5.

d d(y)equation

This block B.12 computes Equation 5.6.

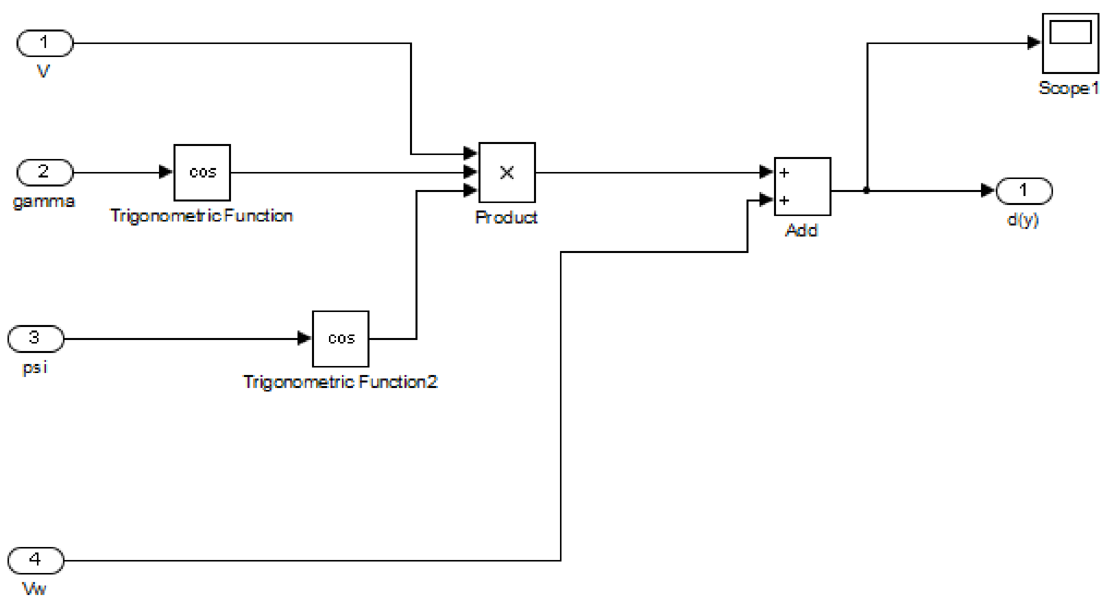


Figure B.12: d(y) equation

- Inputs
    1. V: aircraft airspeed [m/s].
    2. gamma: flight path angle [rad].
    3. psi: aircraft heading [rad].
    4. Uw: East-axis wind speed [m/s].
- Outputs
    1. d(y): derivative of y-coordinate with respect to time.
- Calculations: Equation 5.6

e GS equation

This block B.13 computes Equation 5.18.

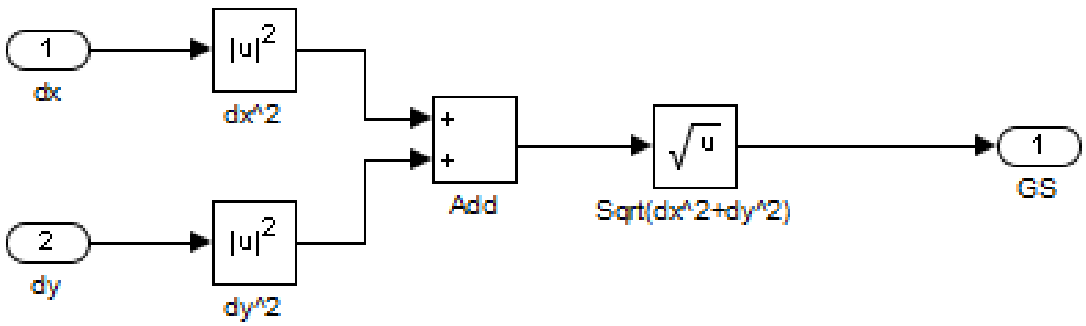


Figure B.13: GS

- Inputs
  1. d(x): derivative of x-coordinate with respect to time.
  2. d(y): derivative of y-coordinate with respect to time.
- Outputs
  1. GS: aircraft ground speed [m/s]
- Calculations: Equation 5.18. Computes the module of d(x) and d(y).

f d(h)equation

This block B.14 computes Equation 5.6.

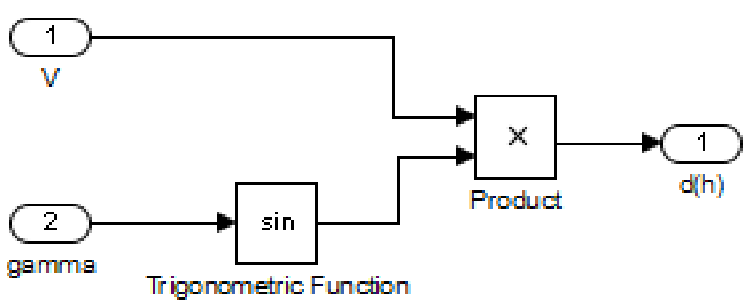


Figure B.14: d(h) equation

- Inputs
  1. V: aircraft airspeed [m/s].

2. gamma: flight path angle [rad].

- Outputs

1. d(h): derivative of altitude with respect to time.

- Calculations: Equation 5.6.

g d(V) equation

This block B.15 computes the derivative of aircraft's airspeed with respect to time using Equation 5.1

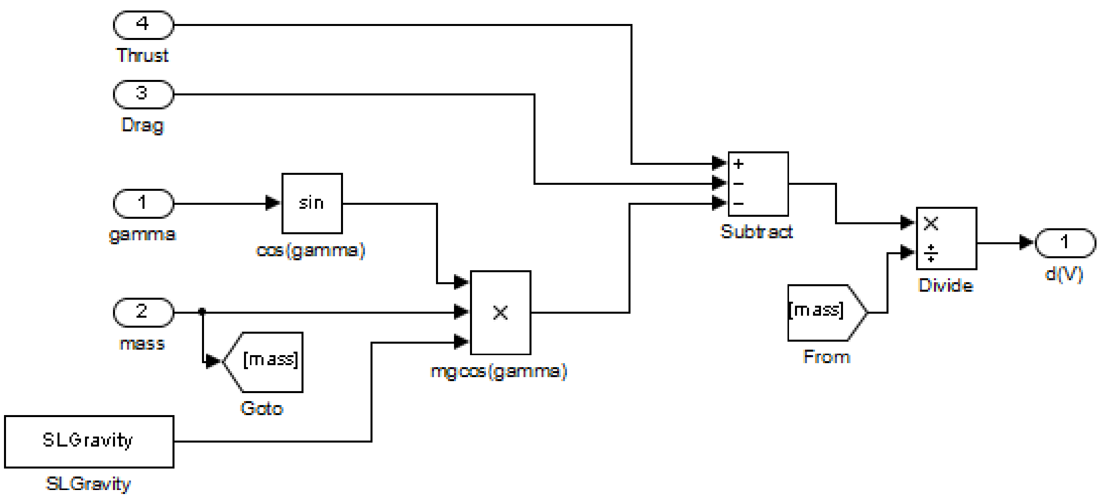


Figure B.15: d(V) equation

- Inputs

1. gamma: flight path angle [rad].
2. mass: aircraft mass [kg].
3. Drag: Drag Force [N].
4. Thrust: Thurst Force [N].

- Outputs
- d(V): derivative of aircraft airspeed with respect to time [m/s^2].
- Calculations: Equation 5.1.

h d(gamma) equation

This block B.16 computes the flight path angle derivative with respect to time using Equation 5.4.

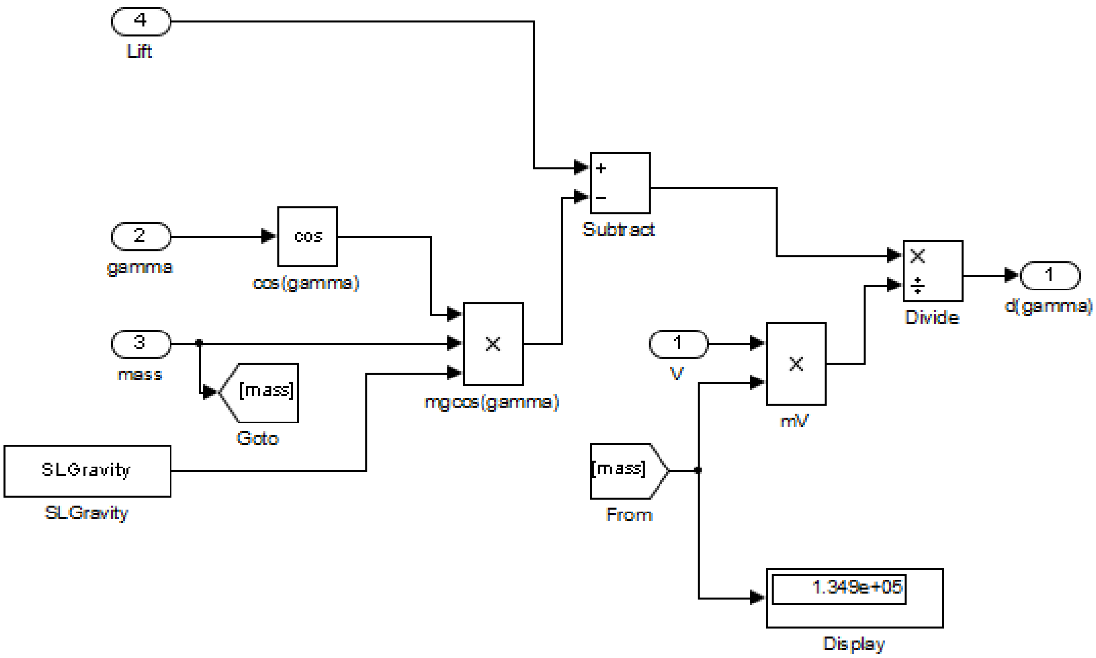


Figure B.16: d(gamma) equation

- Inputs
  1. V: aircraft airspeed [m/s].
  2. gamma: flight path angle [rad].
  3. mass: aircraft mass [kg].
  4. Lift: Lift Force [N].
- Outputs
  1. d(gamma): derivative of flight path angle with respect to time [rad/s].
- Calculations: Equation 5.4.

i d(psi) equation

This block B.17 computes the aircraft heading derivative with respect to time using Equation 5.4.

- Inputs
  1. V: aircraft airspeed [m/s].
  2. mass: aircraft mass [kg].
  3. Lift: Lift Force [N].
  4. gamma: flight path angle [rad].
- Outputs

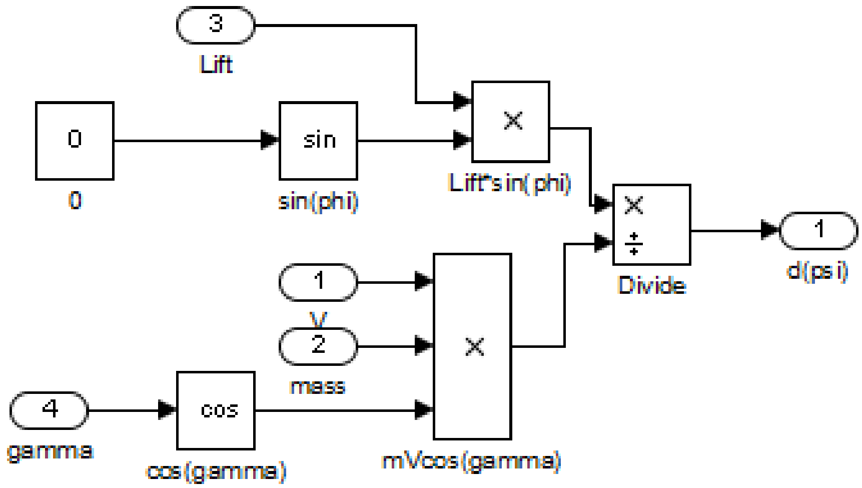


Figure B.17: d(psi) equation

1. d(psi): aircraft heading derivative with respect to time [rad/s].

- Calculations: Equation 5.4.

j d(m) equation

This block B.18 computes the aircraft mass derivative with respect to time using Equation 5.16.

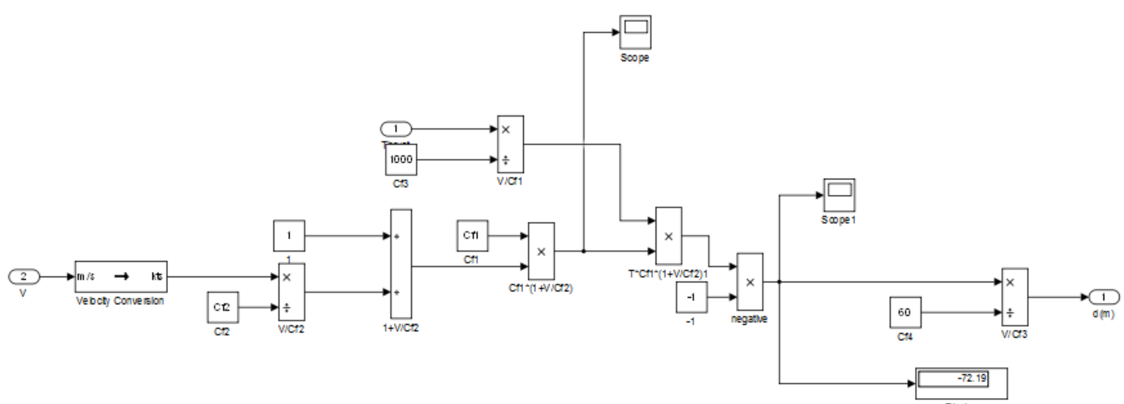


Figure B.18: d(m) equation

- Inputs

  1. Thrust: Thurst Force [N].
  2. V: aircraft airspeed [m/s].

- Outputs

1. d(m): aircraft mass derivative with respect to time [rad/s].

- Calculations: Computes the variation of mass using Equation 5.16 explained in Section 5.3.

B.2.2. Matlab Functions Blocks

k Inside Pre-trajectory block

Listing B.1: Matlab function

```
function [currentPsi, distNextWP, nextWaypoint, currentPsi2] =
 fcn(actualLatitude, actualLongitude, nWaypoint,
  flightPlanLatitude, flightPlanLongitude, V, Uw, Vw)
%% Header
% Author: Joaquim Villen Benseny
% Last Modification: April 4th 2015
% Description: Computes the aircraft's heading in function of the actual
% position, flight plan and wind speeds.
% Reference Document: Bachelor Thesis
% Inputs:
%        -   actualLatitude: current latitude [deg]
%        -   actualLongitude: current longitude [deg]
%        -   nWaypoint: integer containing the next flight
%            plan's waypoint.
%        -   flightPlanLatitude: vector of flight plan's
%            latitude coordinates [deg]
%        -   flightPlanLongitude: vector of flight plan's
%            longitude coordinates [deg]
%        -   V: aircraft's airspeed [m/s]
%        -   Uw: east-axis wind speed [m/s]
%        -   Vw: north-axis wind speed [m/s]
% Outputs:
%        -   currentPsi: aircraft's heading if wind=0 [deg]
%        -   distNextWP: distance to next waypoint [km]
%        -   nextWaypoint: integer containing the next waypoint on
%            the flight plan.
%        -   currentPsi2: aircraft's heading corrected for the wind [deg]

%If reached the destination do not move and go to stop simulation
if (round(nWaypoint)>=length(flightPlanLatitude))
    currentPsi2=0;
    currentPsi=0;
    distNextWP=0;
    nextWaypoint=nWaypoint+3;
else
% Compute distance to next waypoint using Haversine Formula
lat1=pi*actualLatitude/180;
lon1=(actualLongitude)*pi/180;
lat2=(flightPlanLatitude(int8(nWaypoint+1)))*pi/180;
lon2=(flightPlanLongitude(int8(nWaypoint+1)))*pi/180;
R=6371;
delta_lat=lat2-lat1;
delta_lon=lon2-lon1;
a = sin(delta_lat/2)^2 + cos(lat1) * cos(lat2) * sin(delta_lon/2)^2;
```

```
c = 2 * atan2(sqrt(a), sqrt(1-a));
distNextWP=R*c;
% Compute bearing between two points
currentPsi=mod(atan2(sin(lon2-lon1)*cos(lat2),cos(lat1)*sin(lat2)
-sin(lat1)*cos(lat2)*cos(lon2-lon1)),2*pi);

% Compute new heading because of wind
toRad=pi/180;
m=tan(-currentPsi+pi/2);
aEq=(1+m*m);
bEq=-2*(Uw+m*Vw);
cEq=Uw*Uw+Vw*Vw-V*V;
x1=(-bEq+sqrt(bEq*bEq-4*aEq*cEq))/(2*aEq);
x2=(-bEq-sqrt(bEq*bEq-4*aEq*cEq))/(2*aEq);
y1=m*x1;
y2=m*x2;
xWithoutWind=V*sin(currentPsi);
yWithoutWind=V*cos(currentPsi);
if (sqrt((x2-xWithoutWind)^2+(y2-yWithoutWind)^2)<
sqrt((x1-xWithoutWind)^2+(y1-yWithoutWind)^2))
    y=y2;
    x=x2;
else
    x=x1;
    y=y1;
end
heading=180*atan((y-Vw)/(x-Uw))/pi;
if(y-Vw)<0
    if(x-Uw)<0
        heading=90-heading+180;
    else
        heading=90-heading;
    end
else
    if(x-Uw)<0
        heading=90-heading+180;
    else
        heading=90-heading;
    end
end
currentPsi2=heading*toRad;

% If distance to next waypoint less than 2 km go to next waypoint
if distNextWP < 2
    nextWaypoint=round(nWaypoint+1);

else
    nextWaypoint=round(nWaypoint);
end
end
```

I Inside Lift & Drag block

Listing B.2: Matlab function

```
function [CD,alpha] = fcn(alphasVect,machsVect,altsVect,CLMatr,CDMatr,mach,alt,CL)
% %#codegen
```

```
%% Header
% Author: Joaquim Villen Benseny
% Last Modification: April 4th 2015
% Description: Computes the aircraft's drag coefficient by
% obtaining the angle of attack from the lift coefficient previously
% computed and interpolates the drag coefficient in a 3D table. This
% function could be replaced by simulink blocks diagram.
% Reference Document: Bachelor Thesis
% Inputs:
%       -   alphasVect array containing the i angles of attack in
%           degrees used for the DATCOM simulation.
%       -   machsVect: array containing the Mach's numbers used
%           for the DATCOM simulation.
%       -   altsVect: array containing the altitudes in feet used
%           for the DATCOM simulation.
%       -   CLMatr: Set of matrix containing the lift coefficient in
%           function of the angle of attack, mach number and altitude
%       -   CDMatr: Set of matrix containing the drag coefficient in
%           function of the angle of attack, mach number and altitude
%       -   mach: current aircraft Mach's number
%       -   alt: current aircraft's altitude [m]
%       -   CL: current aircraft's lift coefficient
% Outputs:
%       -   CD: current drag coefficient
%       -   alpha: current angle of attack [deg]

% % Vertical dynamics
 coder.extrinsic('clear');

 alt=alt*3.2808399; % Conversion to feet
 % Initailize getAlpha
coefficients=[];
% Interpolation of altitude
equalAlt=false;
for i=1:length(altsVect)
    if altsVect(i)==alt
        equalAlt=true;
        coefficients=CLMatr(:,:,i);
    end
end
j=1;
if equalAlt==false
    if altsVect(1)>alt||altsVect(end)<alt
        error('Altitude out of range');
    else
        while (alt>=altsVect(j))
            j=j+1;
        end
        coefficients=CLMatr(:,:,j-1)+(CLMatr(:,:,j)
        -CLMatr(:,:,j-1))*(alt-altsVect(j-1))/(altsVect(j)
        -altsVect(j-1));
    end
end
alphas=alphasVect;
machs=machsVect;
coefficient=CL;
i=1;
j=1;
```

```
% getAlpha
equal=false;
clVector=[];
for i=1:length(machs)
    if machs(i)==mach
        equal=true;
        clVector=coefficients(:,i);
    end
end
j=1;

if equal==false

    if machs(1)>=mach
        clVector=coefficients(:,1);
    else
        while (mach<=machs(j))
            j=j+1;
        end
        j=j+1;
        clVector=coefficients(:,j-1)+(coefficients(:,j)-
        coefficients(:,j-1))*(mach-machs(j-1))/(machs(j)-
        machs(j-1));
    end
end
alpha=interp1(clVector,alphas,coefficient);

%Altitude interpolation
equalAlt=false;
coefficients=[];
for i=1:length(altsVect)
    if altsVect(i)==alt
        equalAlt=true;
        coefficients=CDMatr(:,:,i);
    end
end
j=1;
if equalAlt==false
    if altsVect(1)>alt||altsVect(end)<alt
        error('Altitude out of range');
    else
        while (alt>=altsVect(j))
            j=j+1;
        end
        coefficients=CDMatr(:,:,j-1)+(CDMatr(:,:,j)-
        CDMatr(:,:,j-1))*(alt-altsVect(j-1))/(altsVect(j)-
        altsVect(j-1));
    end
end
%%

    coefficientFunctionMach=interp1(alphasVect,coefficients,alpha);
    CDefficient=interp1(machsVect,coefficientFunctionMach, mach);
end
end
```

APPENDIX C. WIND DATA HARMONIZATION

The process to obtain the wind data function of the geodetic coordinates is presented in this appendix.

First of all, wind data is provided by the US Aviation Weather Center [Commerce, 2015]. This data is given for 175 airports and 9 different altitudes:

$$3000 \quad 6000 \quad 9000 \quad 12000 \quad 18000 \quad 24000 \quad 30000 \quad 34000 \quad 39000 \quad ft \tag{C.1}$$

How data is provided for one airport is shown in Listing C.1.

Listing C.1: Wind data

```
FT  3000    6000    9000    12000   18000   24000  30000  34000  39000
ABR 1615 2617+05 2918-03 3122-09 3022-22 2928-34 274550 275559 284957
```

The data block shows *ddffttt* where *dd* is the wind direction in tens of degrees, *ff* is the win speed in knots and *ttt* is the temperature in Celsius. At higher levels, the sign for the temperature is not needed since it's assumed all values are negative. If the first digit of the wind direction is greater than 4, then the wind speed is greater than 100. For example, 810550 would be 310º (80-50) at 105 knots and the temperature is -50ºC.

The three first characters of each line are the IATA airport code. Therefore, after reading this data it is required to convert these characters to latitude and longitude coordinates.

This can be performed by downloading a North America waypoint database, for example from [Tomblin, 2015]. In this database there are a lot of different waypoints types with several characteristics as the waypoint name, waypoint type or waypoint coordinates. The complete database has been downloaded. Figure C.1 show all the waypoints in the database.

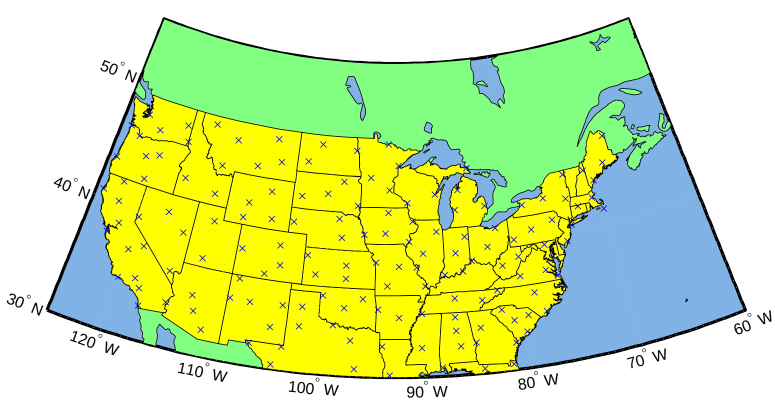
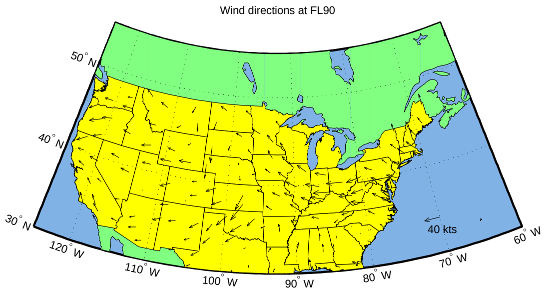


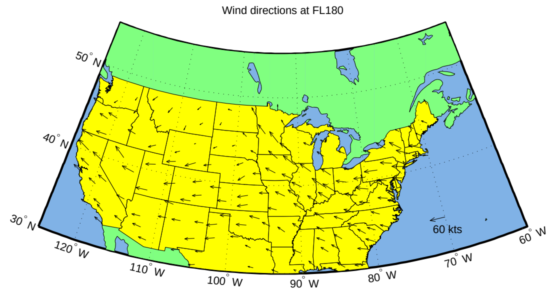
Figure C.1: US waypoints

Then using Matlab, the waypoint names from the waypoint database have been compared to the airport names in the wind database. With this, it has been possible to plot the wind at different altitudes (Figure C.2).

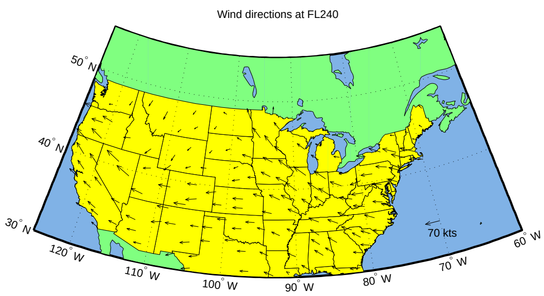
Since this data does not form an uniform grid to ease the interpolation, Matlab has been used to adapt it to a uniform grid using Matlab *griddata* and *meshgrid* functions. The result is shown in Figure C.3.



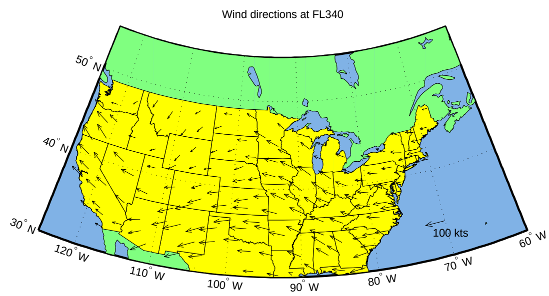
(a) Wind representation at FL90



(b) Wind representation at FL180

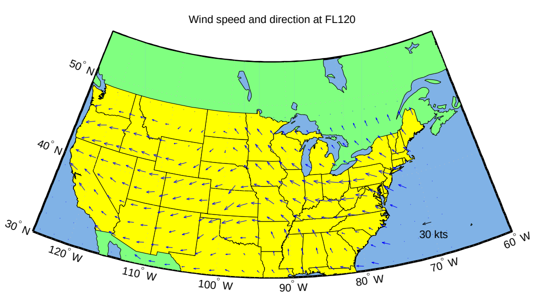


(c) Wind representation at FL240

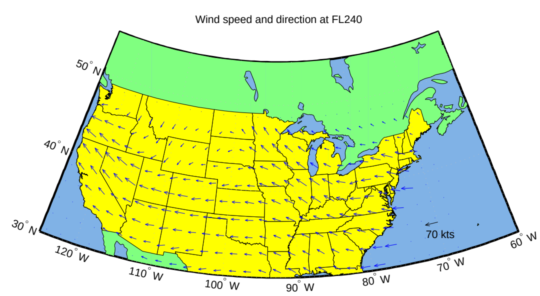


(d) Wind representation at FL340

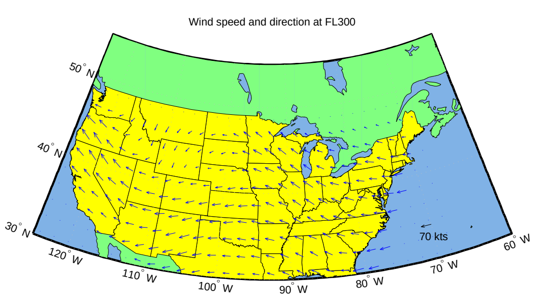
Figure C.2: Wind representation with non uniform data



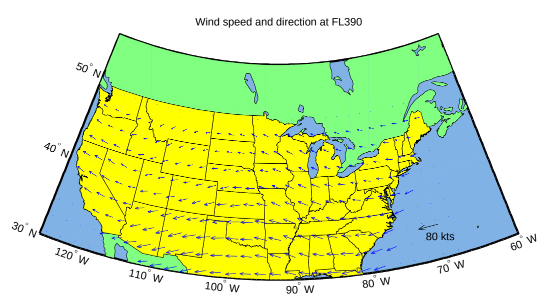
(a) Wind representation at FL60



(b) Wind representation at FL120



(c) Wind representation at FL240



(d) Wind representation at FL340

Figure C.3: Wind representation with uniform data

APPENDIX D. LINEARIZATION

In this the procedure to obtain the drag and lift aerodynamic coefficients during DATCOM simulations is explained.

This is the 3D mass point model presented in Chapter 5.

$$m \cdot \dot{V} = T - D - m \cdot g \cdot \sin\gamma - m \cdot \dot{W}_V \tag{D.1}$$

$$m \cdot V \cdot \cos\gamma \cdot \dot{\psi} = L \cdot \sin\phi - m \cdot \dot{W}_\psi \tag{D.2}$$

$$m \cdot V \cdot \dot{\gamma} = L \cdot \cos\phi - m \cdot g \cdot \cos\gamma + m \cdot \dot{W}_\gamma \tag{D.3}$$

$$\dot{x} = V \cdot \cos\gamma \cdot \sin\psi + W_x \tag{D.4}$$

$$\dot{y} = V \cdot \cos\gamma \cdot \cos\psi + W_y \tag{D.5}$$

$$\dot{h} = V \cdot \sin\gamma + W_h \tag{D.6}$$

The fact that the flight is considered stationary, means that all time derivatives except $\dot{x}, \dot{y}, \dot{z}$ are 0 Then the set of equations D.1 can be rewritten as:

$$0 = T - D - m \cdot g \cdot \sin\gamma \tag{D.7}$$

$$0 = L \cdot \sin\phi \tag{D.8}$$

$$0 = L \cdot \cos\phi - m \cdot g \cdot \cos\gamma \tag{D.9}$$

Then since the cruise phase is considered, the flight path angle, γ, can be set to zero, and the bank angle, ϕ can also be set to zero, since the turns are not considered. Therefore, the set of equations D.7 can be rewritten as:

$$0 = T - D \tag{D.10}$$

$$0 = L - m \cdot g \tag{D.11}$$

The set of equations D.10 show the classic stationary flight where $L = m \cdot g$ and $T = D$. Since the $m \cdot g$ value is known, the lift can be obtained. In order to get the aerodynamic coefficients lift definition needs to be used.

$$L = \frac{1}{2} \cdot \rho \cdot V^2 \cdot S \cdot C_L \tag{D.12}$$

Since L, ρ, V and S are known the lift coefficient C_L can be obtained.

The drag coefficient has been computed in a different way depending on the case.

For the DATCOM simulations since the C_L values obtained with DATCOM depend on α, h and M. Since C_L, h and M are known, it is possible to obtain the value of the angle of attack, α, by linear interpolation. Then, the C_D values obtained with DATCOM depend on α, h and M. Since α, h and M are known, the C_D can be obtained by linear interpolation.

$$W \to L(\rho, S, V) \to C_L(\alpha, h, M) \to \alpha \to C_D(\alpha, h, M) \tag{D.13}$$

The methodology used to obtain the C_D from the C_L for the BADA and Dietrich models is explained in Chapters 3 and4 respectively.